

LEARNING UNDER UNCERTAINTY:
A MODEL-BASED APPROACH FOR
UNDERSTANDING GAMBLING BEHAVIOUR

By


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Thesis Declaration

I, Erica Yu, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

Gamblers in the real world have been found to successfully navigate complex multivariate problems such as those of poker and the racetrack but also to misunderstand elementary problems such as those of roulette and dice. An account of gambling behaviour must accommodate both the strengths and weaknesses of decision making and yet neither of the dominating decision making traditions of heuristics and biases or Bayesian rational inference does. This thesis presents evidence supporting a model-based approach for studying gambling behaviour. The account is built on the premise that decision making agents hold a highly structured mental representation of the problem that is then refined through adjustments made by evaluating incoming evidence. In Study 1, roulette games played at a casino illustrate the range of tactics beyond simple data-driven strategies that are used in chance-based games. In Study 2, an experimental manipulation of the framing of a chance-based dice game highlights the role of prior beliefs about underlying outcome-generating processes. Studies 3 and 4 examine the impact of prior beliefs on subsequent information processing, using a laboratory-based slot machine paradigm. To complement these findings on a computational level, a modelling exercise in Study 5 shows indirectly that assuming a similarity mechanism of judgment is insufficient for predicting the impact of prior beliefs over time. Studies 6 and 7 used racetrack and poker betting experimental paradigms to show that, although priors were integrated into decisions without evaluation, incoming evidence underwent information search and hypothesis and data evaluation processes. Implications for users of gambling research and for future directions of the field are discussed.

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Chapter 1

Introduction

“When the facts change, I change my mind. What do you do, sir?”

(J. M. Keynes, as cited in Malabre, 1994)

While some economists may claim this logic in the face of disconfirming evidence, its sentiment is rarely heard in everyday discourse. Typically, people will go to great lengths to maintain their beliefs and keep their views of the world intact, sometimes deliberately but in many cases without even realising. This inclination toward consistency in policy and reasoning may be at the root of the problem of gambling.

Consider a gambler who has just experienced a string of losses. Keynes might be compelled to change his opinion and walk away from the gamble but the more frequent response is to instead chase the losses, effectively believing anew after each loss that a future win is assured despite evidence to the contrary (Dickerson, Hinchy, & Fabre, 1987; Lesieur, 1979).

This example illustrates several of the motivating questions of gambling research: Why might someone continue to gamble despite losses? Why do only some people have problems with only some forms of gambling? This thesis aims to find what is missing from our current understanding of gambling such that we do not have answers to these questions.

What is gambling?

A gamble, in its broadest sense, is a trade-off between reward and uncertainty: risking a loss for the chance of gaining something greater. From an economic perspective, the elements of this formula are clearly gains, losses, and probabilities and the universal goal of any gambler is to maximise profit. But while a gain of one sort might feel satisfactory, a monetarily equivalent gain of another sort might not. Likewise, an increase in a future risk of a particular magnitude might feel acceptable but another equivalent increase in risk might not (Kahneman & Tversky, 1979). Uncertainty, that is, being unsure about future outcomes because of ignorance about the underlying outcome distribution or

outcome-generating process, leads to inconsistent preferences (Keren & Wagenaar, 1987; Schkade & E. J. Johnson, 1989) and opens us to exploitation. Unsurprisingly, gambles depend on uncertainty.

To understand why people gamble, we can first examine how people gamble. It's unlikely that the punter standing before the bookmaker spends his time pondering over his utilities and risk preferences (though such a discussion of risk preferences in gambling would fill volumes), rather it is highly likely that his mind is making another sort of calculation: what is the outcome of the next event likely to be? To produce an answer involves inductive inference and learning and, perhaps, some specialist knowledge of the event itself because, unlike most experimental tasks, gambles outside of the laboratory are often rich in detail and hidden processes. But, importantly, the tools of inductive inference and learning used in gambling are not different from the tools we use in everyday reasoning.

Gambling as part of the everyday

In everyday reasoning, our decisions are made under uncertainty – we are rarely certain of the outcomes we will see in the future or the processes that will produce them. In processes as fundamental as perception (Freeman, 1994) and as complex as social judgment (Fiedler, 2000) there are parallels: there are infinite theories or functions that could produce the data we observe and so we must make inferences about the environment. Understanding the physical and social world, it has been argued, boils down to a statistical inference problem (Alloy & Abramson, 1979; Kelley, 1967). What would it mean to view gambling as an inference problem? Simply assuming that future outcomes are unknown but that information previously known, rather than observed data alone, may be used to estimate those outcomes. Translated into the more familiar terms found in gambling: inference includes relying on our own previous experience with the game and other games like it, the playing tips received from others, the strategy books and articles read, and the observation of others nearby playing the same game. These data are naturally used as part of the judgment and decision making process in gambling.

Despite the similarities between gambling and everyday reasoning, there is an important difference present in some forms of gambling that sometimes leaves gamblers who are

otherwise competent decision makers vulnerable: randomness. Although randomness is a form of structure, albeit a structure with no systematic bias to speak of, it is qualitatively different from the types of structures we encounter daily. Unlike the movement of clouds, for example, that are driven by hidden but causal processes, the randomness of a card shuffle or die roll is impossible to predict. Nonetheless, this does not stop gamblers from trying.

Perspectives on gambling

To explain how people make inferences in judgment and decision making in gambling, cognitive psychologists have converged on two approaches, which differ greatly in their assumptions and conclusions. Normative decision theory emerged from discussions of rational agents and expected value and utility. Heuristics and biases came from a need to describe the actual behaviour that researchers observed. The following pages review the theory and impact that each approach has had on the study of gambling.

Normative decision models

Normative decision theory identifies the best decision to make, assuming the decision maker has complete information and reasons rationally about the outcomes and their likelihoods. Players have been found to use normative theory as a playing strategy in blackjack (Wagenaar, 1988) and poker (Sklansky, 2004), among other games. For this discussion of a normative standard in gambling, there are two relevant approaches.

A first account, dating from Huygens in 1657 laying out probability theory and Von Neumann and Morgenstern (1944) introducing utility theory, came to be viewed as a model of human decision making. Decision makers need only to calculate a probability-weighted sum of the possible outcomes to determine the value of a gamble. Games with dice illustrate the straightforward use of a normative decision rule to calculate the value of gamble. If a bookmaker offered a punter a gamble for which the punter paid 25 pence for the opportunity to roll a die and earn £1 if he predicts the number correctly, an expected value calculation finds the terms are unfavourable: the punter has a 1 in 6 chance of guessing the outcome correctly and gaining £1, so the expected value of his 25 pence is only 17 pence, which is an expected loss of 8 pence for each play. And, as Savage (1965) points out, over repeated plays that are sure to include both wins and losses, the expected value of bad compounded odds does not get better.

A second normative model for gambling, more recently developed by Edwards and colleagues (1961; Edwards, Lindman, & Savage, 1963), brought Bayesian statistical inference to the field of decision making. Bayes' theorem is a general mathematical rule that is commonly used for belief updating using evidence that results in a posterior probability expressing the degree of belief about the likelihood of a hypothesis being true after observing data. The probability of a hypothesis based on evidence is a function of the probability of the hypothesis generally, the probability of the data generally, and the likelihood that the data were predicted by the hypothesis. The fundamental principle underlying this logic is the competition between hypotheses, and Bayes' theorem favours the hypothesis that is most likely relative to others. Griffiths, Kemp, and Tenenbaum (2008) demonstrate an application of Bayesian inference on determining whether a fair coin (a hypothesis of the probability of the coin landing on heads of 0.5) or biased coin (an alternative hypothesis of, for example, a probability of landing on heads of 0.9) produced a series of coin flips. Imagine initially believe that either hypothesis is equally likely to be true but then observing HHHHHHHHHH. According to Bayes' theorem, this sequence of evidence changes the likelihood that the coin is biased from even to 357:1. If instead the sequence HHTHTHTTHT had been observed, the likelihood that the coin is fair would change from even to 165:1. Bayesian inference does not produce conclusive answers but does provide a systematic way to evaluate evidence. Instruction on using Bayes' theorem can be found in poker strategy books and Internet forums (as of this writing, more than 5,460 hits on Google for "Bayes' theorem, poker") for applications such as evaluating the likelihood that an opponent holds a viable hand given his actions¹. It is also being used in artificial intelligence to model learning against simplified players (Korb, Nicholson, & Jitnah, 1999).

The normative models of gambling have played a central role in shaping our beliefs of what a rational gambler might be expected to do. Expected value as a threshold between rational and irrational gambling is now a universal concept. From this reference point, researchers have found deviations, clinicians have set therapy expectations, and policymakers have set industry standards. But how many would defend the normative model as a description of actual gambling behaviour?

¹ However, not all instructions are accurate, likely reflecting a widespread misunderstanding of the theorem in the amateur poker playing community.

Beyond rational models?

There are significant limitations of the applications of normative decision theory as a useful account of gambling behaviour. Although it is useful in its own right as a prescriptive account of how one ought to gamble, there are several demonstrations of its descriptive inadequacy. For example, Kahneman and Tversky (1979) showed that people treat gains and losses differently from their objective values. It is widely accepted that the assumptions of Bayesian inference are ideal at best, as the imagination and cognitive capacity required to consider all evidence even-handedly, generate exhaustive sets of hypotheses, and calculate likelihoods are out of reach for most people, including experts (Fischhoff & Beyth-Marom, 1983; Fischhoff, Slovic, & Lichtenstein, 1978; Meehl, 1954). Probability and variance judgments are not consistent (Slovic & Lichtenstein, 1968). And in a real-world gambling context, Keren and Wagenaar (1985) show that blackjack players deviate from the normative basic strategy, seem to be inconsistent in what strategy they do use, and believed that “luck” played a significant role in generating outcomes.

Beyond using the rational model as a reference point for ideal decision making agents, what can the clinician or policymaker use it for? The mechanisms underlying the hypothesis generation and updating process are impenetrable to a clinician seeking to understand individual behaviour.

In a critique of the Bayesian perspective, Fischhoff and Beyth-Marom (1983, p. 257) note that a “substantive understanding of the problem at hand” is necessary to accomplish many of the operations assumed by Bayes’ theorem and normative decision making generally. Generating the possible hypotheses and prior knowledge relevant to the problem are not trivial matters. Continuing in this direction, consider that every problem has a higher-order structure underlying its outcomes. There are interdependencies and causal processes linking stimuli, actions, and rewards that we intuitively use for judgments, informing how we interact with the physical world and with each other. A complete account of gambling behaviour and cognition must acknowledge the roles of structure and outcome-generating processes.

The heuristics and biases approach

Heuristics are strategies for producing estimations and predictions based on intuitive assessments rather than deliberative algorithmic processing. This intuition-based approach embraces the idea of gambling decisions as a subset of everyday reasoning. The three original heuristics introduced by Kahneman and Tversky (1974) are representativeness (judging the likelihood of a hypothesis based on how closely it resembles the data), availability (judging the frequency of an event based on how easily an example comes to mind), and anchoring (relying too heavily on one piece of information). Despite being grounded in intuitive rules-of-thumb, the heuristics are not completely removed from the rational model as they are built on fundamental concepts of cognition like feature matching, similarity, and memory retrieval. For example, subjective probability estimations are similar to subjective estimations of physical quantities such as distance, whereby the distance of an object is not calculated but intuitively judged by its blurriness (far) or clarity (near).

Consider representativeness, which is critical for a discussion of gambling as it is often cited as the source of likelihood judgments. When a likelihood judgment is mediated by an assessment of similarity to its parent population or reflects the process by which it is generated, decision makers are said to be using the representativeness heuristic. For example, sequences that include all possible outcomes are judged more likely than sequences that exclude any outcomes even if those excluded are highly unlikely, presumably because the sequence that includes all outcomes is deemed more similar to the parent population (Kahneman & Tversky, 1972). Consider the coin flip, a random binomial process. The researchers found that a sequence of coin tosses that contains either obvious regularities or excludes some possible outcomes is not considered random. Overall, a sequence that is not representative of a random sequence is deemed non-random, or structurally biased. In other words: predictable. This belief in the negative or positive recency of a sequence of random events has been called, in turns, the gambler's fallacy (believing that the likelihood of an independent event occurring increases after an absence) and the hot hand (believing that the likelihood of an independent event occurring increases after a streak), and deemed responsible for many of the problems of reasoning exhibited by gamblers.

As in the example above, the heuristics approach is commonly applied to the study of gambling for explaining how gamblers irrationally deviate from rational choice and the probability calculus. A few examples include Rachlin (1990) applying availability to losses, Gilovich (1985) applying representativeness to streak shooting, and Rogers (1998) applying anchoring to lottery judgments. And beyond the original cognitive heuristics identified, there are also now the affect heuristic (Finucane, Alhakami, Slovic, & S. M. Johnson, 2000) and the somatic marker hypothesis (Damasio, Everitt, & Bishop, 1996), which substitute affect and emotional processes for intuitive thinking.

In the domain of gambling, probability judgments and predictions are so universal that heuristics for explaining such judgments are powerful and amount to explanations for much of gambling behaviour. In gambling research, heuristics have become synonymous with erroneous perceptions and distorted cognitions and are implicated in a range of literatures beyond psychology. Indeed, the heuristics approach has changed the way that clinicians treat patients and envision therapies (Blaszczynski & Silove, 1995; Delfabbro, 2004; Toneatto & Sobell, 1990; Zangeneh, Blaszczynski, & Turner, 2007) and politicians develop policies (Eggert, 2004).

Going beyond heuristics?

Despite its strengths, the heuristics programme of research has its critics. Does the research showing participant blunders demonstrate that people are poor at inductive reasoning or that the researchers and their experiments are poor at eliciting accurate responses (Cohen, 1981; Gigerenzer, 1991)? Are the experimental tests provided as evidence just illusionary circumstances lacking ecological validity (Lopes, 1982)? Might participants be simply satisficing (Nisbett & Ross, 1980)? Does the research hold people to a reasonable standard (D. F. Barone, Maddux, & C. R. Snyder, 1997)? For a comprehensive review of criticisms written by two of the field's strongest proponents, see Gilovich and Griffin (2002).

But the question of interest for this discussion is whether the heuristics approach is appropriate for the study of gambling. Even one of its proponents of applications to gambling, Wagenaar (1988), notes that the harsh conclusions drawn of doomed irrationality are sometimes unwarranted and asks why, if in everyday reasoning a heuristic is considered sensible, is the same reasoning in gambling deemed irrational? He is joined

by Corney and Cummings (1985) who also posit that the so-called gambling biases are simply the biases of everyday information processing. There is no evidence that irrational thinking is related to excessive gambling as the biases are just as prevalent in non-gamblers as they are in regular gamblers (Delfabbro, 2004; Ladouceur & Walker, 1996). And, simply put, gamblers staking large amounts of money have every reason to think deliberately rather than intuitively.

Referring back to the agenda stated at the start of this chapter, the field of gambling research seeks to address the question of why people continue to gamble despite losses and negative expected value. Their answer appears to be simply that erroneous beliefs persist, paying no heed to the evidence that people exhibit learning and successful decision making in other domains. And why do gambling problems affect some but not all? A core principle implicit in the heuristics and biases account is that people deviate from rational choice in predictable and systematic ways (Gilovich & Griffin, 2002, p. 1). But in some instances there is not even a clear formal concept of what the heuristics are or what they predict or when one heuristic might apply to a problem rather than another; for example, “representativeness” (Gigerenzer, 1991). The heuristics and biases programme has predictability only in hindsight.

Model-based gambling

A model-based account of decision making applied to gambling behaviour would provide a useful and descriptively valid account of decision making. In brief, the model-based account allows for an agent to hold rich structured knowledge about the environment and develop ideas about the hidden processes generating observed outcomes, which are both updated by integrating prior beliefs with incoming evidence. Such an approach builds on established findings in learning based directly on reinforcement learning and also indirectly on schemas and mental models.

Reinforcement learning

In early conceptions of reinforcement learning, such as those used originally to describe trial-and-error learning in animal behaviour, an agent interacts with his environment through perception (observing stimuli) and action (responding to stimuli), assuming no role for cognition (Kaelbling, Littman, & Moore, 1996). Importantly, a consequence of this structure is that while the agent observes outcomes in the environment, he does not

observe or at least does not record knowledge of the process that produced the outcome. The outcome merely adjusts a scalar reinforcement signal (e.g., reward) without evaluation that is then used to determine the best next action. This approach grounded in statistics has a long history with gambling, including the problem of and solution for optimal decision making among slot machines as discussed by Berry and Fristedt (1985). Modern theorists now refer to this version as model-free learning.

Model-based reinforcement learning (Chater, 2009; Sutton & Barto, 1998; Sutton, 1991), in contrast, is intrinsically concerned with cognition². It has two premises: the agent builds a “model” or explicit internal representation of how the environment works and he chooses the action that is best conditional on his current knowledge. This agent is able to learn not only about the relationships between outcomes and his own actions but also states, or different environments. The model may be influenced by all and any knowledge that the agent has, including the history of past actions, outcomes, and states. This model amounts to being able to access from memory how to play the game of poker, strategies for play and how to implement them, the previous experiences of playing against one’s present opponents, and the second-hand information received from others about playing against one’s present opponents. How the model is updated is not strictly defined; however, because information is evaluated and retained, adjustments to models can be forward and backward looking as well as global (across multiple domains) or localised. The model also has the components of structure, properties, and relations that organise an environment required to make causal inductions (T. L. Griffiths & Tenenbaum, 2005). Such an account is far-reaching but also simple enough to cover a range of situations under uncertainty.

Schemas and mental models

To extend the notion of and provide additional support for the psychological validity of the model-based approach, a cursory background on the concept of a model or internal representation of an external event or idea is warranted. Schemas are structures of knowledge that include concepts of components, attributes, and relationships between specific instances (Bower, 1981; Pearson, Barr, & Kamil, 1984). Schemas have been explored and documented in a range of domains, including person stereotypes and roles,

²This may be debatable, between the rational and mechanistic perspectives on reinforcement learning (see Chater, 2009). However, in this thesis and for the purpose of a multi-disciplinary application to gambling research, only the rational approach will be considered here.

goal-oriented action sequences, and narratives (for a comprehensive review, see Graesser & Nakamura, 1982). Similarly, with mental models, it is argued that people use an internal symbolic understanding of the external world. People can only represent small-scale models of reality, neglecting facts and relationships that are outside their scope of knowledge with cascading effects on reasoning (Johnson-Laird, P. Legrenzi, Girotto, M. S. Legrenzi, & Caverni, 1999).

The premise underlying these approaches is that cognition is a top-down process, whereby attention, interpretation, general impressions, and in-depth reasoning are all driven by structured models. Gestalt psychologists argued that the properties of an experience could not be inferred from its parts alone; some properties are inferred from the prior knowledge of the part. Murphy and Medin (1985) argued that simple feature models based on similarity are ineffective as our conceptual systems rely on explanatory, causal, teleological and ontological beliefs, such as those represented by top-down processing. For example, when reading a story in which the protagonist dines at a restaurant, one might not read the explicit description of the customer paying his bill but one infers that he has (Pearson et al., 1984). A schema or mental model is the highly-ordered structure in which the rich knowledge about the game of poker and the typical strategies used by other players is stored.

At first, the model-based approach may seem to be a Bayesian in sheep's clothing. The formula of prior beliefs integrated with evaluated incoming evidence to determine the best next action sounds familiar. Indeed, there are examples of model-based learning that use Bayesian algorithms to update models (Dearden, Friedman, & Andre, 1999). But the differences are significant. Where in Bayesian approaches evidence is used to update a distribution of beliefs or hypotheses about the environment, model-based approaches update only one or few beliefs about the structure of the environment, within the known bounds of cognitive capacity limitations (Doherty & Mynatt, 1986). The updating process is also qualitatively different: the model-based learner evaluates incoming evidence to adjust his current model of the environment rather than globally assess all observed evidence. Consequently, posterior beliefs may not reflect all evidence observed but instead be biased by effects of presentation order and framing. A closer look in the following section will clarify any confusion.

The elements of a model-based account

The model-based approach to gambling behaviour is built on subjective internal representations of the problem and the agent's interactions with the environment. It is interested in answering the question of *how* people gamble. It can be depicted as a feedback process whereby the outcomes of the agent's actions are fed back into the agent's beliefs about the problem. As more evidence is collected, the agent may confirm or adjust his belief about the underlying outcome distribution and outcome-generating process. The approach may be as simple as a lookup table or as complicated as a search process (for dynamic problems, the agent can learn the transitions between states). At all levels of complexity, there are two principle components:

A model, schema, or prior belief about the problem—The agent has a structured representation of the problem that reflects the rich prior knowledge and intuitions brought to the task. The model includes representations of the states of the environment, actions, and rewards and the explanatory, causal, teleological and ontological beliefs about the relationships among them; for example, how actions are related to rewards. The top-down structure enables the agent to reason broadly about classes and categories of events and relationships using prior knowledge as well as in a more detailed manner about the specifications of the current problem using data.

A belief-adjustment process—Upon observing new evidence, the agent evaluates the information against his prior beliefs, regardless of whether the information was collected through active information search or passive observation. Because the prior beliefs contain only knowledge and experience available to the agent, the evidence does not impact unknown hypotheses. And, as a consequence, learning can be biased by order effects. However, because models of the environment are highly structured and include histories of previously held beliefs, new information can go beyond adjustments to cause qualitative and systemic changes in models. For example, verbal instructions of task changes may be immediately implemented without trial-and-error learning (e.g., Bridger & Mandel, 1965), retroactively to previous beliefs, and broadly resulting in all-or-nothing learning (Gallistel, Fairhurst, & Balsam, 2004).

Critically, as a consequence of these two dissociable components, an agent may appear to behave irrationally while implementing rational inference. If the agent's model of the problem is inaccurate, it may be the case that new evidence does not correct the errors

despite the correct implementation of the updating process. The agent may persist in believing inaccurate information and acting sub-optimally in the face of disconfirming evidence.

Consider an example. Upon encountering a game, the decision making agent immediately draws upon prior beliefs to implement his first action. This may include intimate knowledge of the game structure or just best-guesses based on environmental cues available. The outcome of his action is then fed back to him, and used to update his beliefs about the problem he is facing and his best next action, perhaps in the form of ‘Since 29 was the winning number, I just need to tweak my system a little to make it work.’ Such a system of beliefs accommodates the types of verbalisations reported in observations of gamblers, including beliefs about luck and skill. And because the model is a subjective internal representation of reality, these beliefs may be held regardless of the true chance or skill nature of the game.

Summary

The field of gambling research from a psychological point of view has been dominated by the heuristic and normative points of view. However, neither approach to gambling adequately captures the strengths and weaknesses of actual real-world gambling behaviour nor provides users of gambling research with a practical means of understanding gamblers. Instead, a model-based approach, which puts the individual at the centre of a rational process, may bring a new perspective to the field. The following chapters will cover gambling tasks that range from simple to complex and use stimuli from both the laboratory and the real world to showcase how a model-based account improves upon the current thinking of the field. The experimental paradigms and discussions will expand upon the ideas introduced here, about model-based decision making, models, and updating processes. In every instance, understanding how the individual thinks about a gamble will be emphasised. This thesis aims to demonstrate that, by applying a model-based learning approach to gambling, we may advance the field in new directions.

Outline

Classes of Uncertainty

Much of the discussion about games is centred on the evaluation of outcomes, by academics and punters alike. Questions often raised are ‘How many times did it come up red?’ or ‘How many times did the individual continue to bet despite losing?’ But, often times, this focus is misguided because of the lack of control over outcome-generating processes, such as in games with chance elements. Both groups of academics and punters ought instead to be focused on the processes that produce the outcomes. The nature of this process informs the predictability of the game’s outcomes and the rationality of a wagering endeavour. This extends to all types of gambles, from the simplest to the most complex. In the following sections, I divide uncertainty into three classes in which decision making agents do not or cannot assign objective probabilities to actions and outcomes: uncertainty with static and known probability distributions, static but unknown probability distributions, and dynamic processes. Each is described from the established cognitive psychology perspective with examples from games found in the literature and casinos.

Fixed and known probability distributions

In a well-defined outcome space of fixed and known underlying probabilities, outcomes are typically considered as risks. Although the outcome-generating process may be random, decision making agents faced with risky choices have full information regarding the possible outcomes and the likelihoods of these outcomes. There are important differences between games such as these played only once or played repeatedly over time. In those played over time, because the outcome-generating process is static over time, agents can observe evidence and learn about the processes.

Classically, economists and psychologists have represented risk with simple gambles, such as the choice between two options (Kahneman & Tversky, 1979, p. 264):

- A: 50% chance to win 1,000, B: 450 for sure.
50% chance to win nothing;

Within a gamble, outcomes are yielded with static and specified probabilities, such that the probabilities of each outcome sum to 1, indicating that all possible outcomes are known.

Consider the risky gambles offered at a European roulette table. A win is determined by the resting position of a ball spun on a wheel; the outcome is the product of a random process. The wheel is divided into equal-size areas labelled with 37 numbers, giving each area (number) an equal probability of winning. In the format of the gamble above, this may be expressed as each number occurring with a probability of 1 in 37 or 0.0270. The bets offered at a roulette table range from single numbers (e.g., 24) to large portions of the wheel (e.g., even numbers) that have known and calculable odds of winning. In principle, roulette offers straightforward risks that people understand.

From the normative position, this straightforward risk of a one-shot gamble has been proven to extend to repeated gambles: it can be shown mathematically that the unfavourable odds of a gamble are compounded for repeated plays, such that a gamble with unfair odds simply remains unfavourable under repeated play (Dubins & Savage, 1965). Empirically, however, an understanding of straightforward risk over a sequence of decisions is often quickly confused by decision making agents. People indicate preferences for riskier, lower expected value options when they know they will gamble repeatedly, compared to playing single gambles (Coombs & Bowen, 1971). In trying to predict or control outcomes, people transform the straightforward proposition of repeated risky gambles into a complex learning problem.

One way by which people transform gambles into more complex problems is by putting aside the well-known principles of randomness and probability and instead relying on previously seen outcomes. People seek—and find—patterns in the data. For example, in a game lasting over many roulette spins, a curious punter may find that streaks of reds are followed by a black number. Although this pattern may be genuine, it is only a logical necessity of the binary nature of the outcome space (aside from the green zero, all numbers are either red or black) and does not provide any power in predicting randomly-generated outcomes. Analysis and discussion of data collected from electronic roulette machines in a land-based casino and lab experiments will further explore this concept of

learning under uncertainty with static and known underlying probability distributions in Chapter 2.

Fixed and unknown probability distributions

While a gamble's outcomes may be known, as in the simplest class of uncertainty problems described above, there are times when the probability distribution underlying those outcomes is unknown. This type of uncertainty is often described as ambiguity. Decision making agents facing ambiguous gambles do not have full information about the likelihoods of outcomes. Effectively, this type of knowledge is analogous to ignorance of the outcome generation process—its causal structure, as one example. However, given the static nature of the problem, it is theoretically possible to approximately learn the probabilities.

In psychology and economics, this type of gambling under uncertainty has often been presented in urn experiments. A seminal gamble premise is re-created here (Ellsberg, 1961, p. 650):

Let us suppose that you confront two urns containing red and black balls, from one of which a ball will be drawn at random. To “bet on Red₁” will mean that you choose to draw from Urn I; and that you will receive a prize a (say \$100) if you draw a red ball... and a smaller amount b (say \$0) if you draw a black... Urn I contains 100 red and black balls, but in a ratio entirely unknown to you; there may be from 0 to 100 red balls. In Urn II, you confirm that there are exactly 50 red and 50 black balls.

Within an ambiguous gamble, the relevant probabilities are unknown. In this example, the decision making agent has no information about the likelihood of drawing a red or black ball from Urn I, in contrast to the full information about the likelihoods for Urn II (which is an example of the class of uncertainty with static and known probability distributions).

Consider the gamble offered by slot machines, fruit machines and poker machines. These machines typically offer a video interface of five reels of symbols that spin and come to rest showing combinations of symbols. A spin's payout is determined by the matching of symbols on the played payout lines to the machine payout table's set of winning

combinations. For example, if there are three cherry symbols along a payout line, the punter receives the prize associated with that combination. These machines offer a limited set of winning combinations; all possible outcomes and winning combinations are known but no probabilistic information is available. In the example given, the likelihood of three cherry symbols appearing along a payout line is unknown. However, the unknown likelihoods do not change over time. In principle, these likelihoods can be approximately learned and this class of uncertainty gambles can be transformed into risky gambles.

This class of gambles offers another hook that increases subjective uncertainty when considered over time: with unknown underlying probability distributions, false probability distributions can be imposed on the gamble. Regardless of any misguided belief in illusory correlations based on circumstance or skill, the likelihood of outcomes can be incorrectly judged. A combination of small stakes, infrequent and salient prizes, and long playing sessions may result in an exaggeration of the frequency of winning; for example, the common phenomenon of duration neglect (Fredrickson & Kahneman, 1993; Kahneman, Fredrickson, Schreiber, & Redelmeier, 1993), whereby one might underestimate how long one has been playing or how many small bets or games one has played, may result in a belief that large wins occur more frequently than is true. This exaggerated likelihood effectively increases the expected value of taking the gamble. Analysis and discussion of data collected from lab experiments using simulated slot machines and a modelling study will further explore this concept of learning under uncertainty with static and unknown underlying probability distributions in Chapter 3.

Dynamic processes

Gambles with dynamic underlying processes represent the most complex class of uncertainty and are qualitatively different from the two presented previously. The outcome space may be undefined and the outcome-generating process may be unknown and variable at the time of judgment. There are no mathematical calculations for determining optimal judgment and decision making. This type of gamble is less commonly found in the gambling research literature but is common in everyday life.

Consider handicapping a horse race where the goal is to predict the post-time odds of the top-finishing horses. Before the race begins, a variety of information is available, including historical statistics on each horse's background (e.g., breeding, weight, price)

and recent race results (e.g., the purse size of previous races, number of pass attempts). However, some other variables have significant effects on the outcome of the race but cannot be quantified, such as the horse's personality. And, actual race results may be partly determined by chance (e.g., unforeseeable collisions).

Ceci and Liker (1986) studied the decision making processes of expert and non-expert handicappers using both actual race information and results and experimentally-designed hypothetical races. The most significant finding is the characterisation of punters' cognitive process of modelling the race: seven or more factors about the horses' previous races are considered, in a process approximating multiple regression with multiple interaction terms. Each piece of information is related to another piece, and understanding horse racing requires understanding how the information is structured. For example, a judgment about the objective record of a horse's final speed is qualified by the quality of its competitors, the number of pass attempts, its starting position, and more. Analysis of the punters' verbalisations indicates that schemas are used to create a probabilistic construction of the race. Indeed, the skill in the creation and use of accurate schemas may be what separates experts from non-experts with equivalent experience.

Although a dynamic outcome distribution cannot be learned with certainty, an ideal learner may eventually predict outcomes with more accuracy than by chance or simple accumulation of data. Using inferences and hypotheses about the outcome-generating process, learners may reach beyond the data to a deeper understanding of the underlying processes. Analysis and discussion of data collected from lab experiments using horse race and poker game scenarios will further explore this concept of learning under uncertainty with dynamic underlying processes in Chapter 4.

Chapter 2

Fixed and Known Uncertainty

Consider a coin flip. We characterise it as the simplest of processes: throw a coin into the air and observe the outcome as either heads or tails with precisely equal chance. Flipped once or flipped a thousand times, the likelihood never changes. We rely on it as an arbiter of fairness for decisions that we would not leave to our own emotions and biases. But is the outcome of a coin flip genuinely without bias? The initial position of the coin, its momentum and its environment are all parameters that complicate the process in reliable ways (Keller, 1985). The physical deterministic process of the coin flip may result in equal likelihoods of heads or tails on average but each has its own biases. Even a coin flip is not as straightforward as it seems.

How do decision making agents consider gambles on coin flips? This problem is a subset of gambles under fixed and known uncertainty. Available to the agent is complete knowledge of the sample space and the static outcome-generating process. While it is clear that the agent has the knowledge of the underlying process in this case, it is less clear whether he obtains the normative understanding of the process. Effectively, agents may be treating these gambles as more complicated problems of uncertainty with *unknown* underlying outcome-generating processes. For example, is the roulette calibration slightly biased? Is a streak of outcomes in a quadrant of the wheel evidence in support of a biased wheel or just a random occurrence? Interpreting the formal problem and processing evidence are two points at which agents might diverge from the normative understanding and from each other.

Interpreting the problem

According to probability (rational) theory, probabilities of events can be inferred in an extensional way: the probability of a set of events occurring is determined by its members and therefore the judged likelihood of a set of events should be the same whether elicited as a whole or in parts. However, it has been shown that the extensionality assumption of rational probability theory does not hold true: different framings of a formally identical problem change how decision makers judge likelihoods and make choices (Tversky & D. J. Koehler, 1994).

This finding is simply demonstrated by Johnson and colleagues in an experiment in which the researchers offered hypothetical hospitalization insurance for *any disease or accident* or *for any reason*. (E. J. Johnson, Hershey, Meszaros, & Kunreuther, 1993). The latter option – *any reason* – is the broadest possible coverage option and dominates the former option that names only disease or accident. Despite this, participants offered the inferior option were willing to pay more than those offered the superior option. This phenomenon is also found in the so-called “law of small numbers” whereby people expect small samples from random sequences to reflect the overall proportion of outcomes from the parent sequence, or, in other words, to be representative (Kahneman & Tversky, 1971). In both of these examples, decision makers do not obtain the correct understanding of the problem described.

Another way to diverge from assumed normative behaviour is to misinterpret the underlying randomising process. Implicit in an understanding of a game on offer is an understanding of the underlying process that produces the outcomes. One example of this is when people believe they have control over outcomes despite the game being uncontrollable. In her (1975) paper on the illusion of control, E. J. Langer proposes that this bias is based on the confusion between skill games that have a causal link between action and outcome and chance games that decidedly do not. In Study 1 of E. J. Langer (1975), participants engaged in a task in the presence of a confederate posing as another subject. The task to be completed was a game between the two players based on card draws with the winner of each round determined by the value of the card drawn; in other words, the outcome of each round was random with no skill involved. Before each round of card draws, each participant was asked to privately record a wager between 0 and 25 cents on whether he would win that round’s game. The experimenters manipulated the appearance of the confederate as either *dapper* (competent appearance) or *schmook* (incompetent appearance). As hypothesised, participants wagered significantly less when “competing” against a competent person than against an incompetent person. E. J. Langer (1975) also demonstrates this finding with other skill-related factors such as choice, stimulus or response familiarity, and passive or active involvement. Factors of the descriptions of the problem with no bearing on the value of a gamble can have significant impacts on behaviour.

Learning from random outcomes

Stripping an experimental task of any resemblance of skill though does not necessarily improve decision making. In a simple repeated binary choice probability learning task, people demonstrate in another way their preference for structure and meaning in outcomes: probability matching. Despite rewards being dependent on the accuracy of choices, people tend to make choices that match the choices' probabilities of occurring and consequently perform poorly (Neimark & Shuford, 1959). As Tune (1964) notes about these findings, participants seem to prefer patterns.

Chance-based games are a special case of decision making tasks: there are no meaningful patterns or structure to be found. Despite the knowledge that an underlying structure is random, participants asked to produce sequences such as outcomes of flips of a fair coin consistently produce biased responses. Participants try to “match” their representations of randomness and produce over-alternation and locally representative sub-sequences (Falk & Konold, 1997; Gaissmaier & Schooler, 2008; Lopes & Oden, 1987; Rapoport & Budescu, 1997; see Nickerson (2002) for a comprehensive review of the literature).

Particularly relevant to gambling are two biases in randomness perception identified in the literature as the gambler's fallacy and hot hand. The gambler's fallacy is the belief that chance processes monitor themselves and balance out runs of particular outcomes and is common in roulette, slot machine, and other table game players (Delfabbro, 2004; Walker, 1992). A run of heads increases the likelihood that the alternative outcome, tails, will occur. This belief in negative recency is in direct contrast to a belief in positive recency, known as the hot hand. The hot hand is a belief that a run of heads increases the likelihood that the same outcome, heads, will occur. For example, after a streak of basketball shot successes, fans, coaches, and players alike believe that another success is likely to occur next, though the true likelihood of a success is just 50% and is independent of previous shots (Gilovich, Vallone, & Tversky, 1985; although see also Sun (2004) for arguments that these processes are non-random).

In the examples described above, decision makers fail to understand the options and the terms of the gambles offered despite both being clearly expressed and all information available. Contrary to normative assumptions, known uncertainty is evidently quite

complicated³. People's judgments are biased by framing, structure and meaning are imposed on randomness, and the same sequence of outcomes can be used to support two opposing predictions of what will happen next. The heuristics tradition's approach to these problems lacks a coherent complete account – representativeness is recruited to explain both the gambler's fallacy and the hot hand with no unifying theory to a priori indicate which. Neither rational choice nor heuristics are able to explain these findings.

Model-based accounts take a different approach. Because we are concerned with fixed and known uncertainty in this instance, these beliefs are deemed erroneous by the normative standard, but in domains other than chance where events are not independent or uncontrollable, such beliefs may not be fallacious at all but rather adaptive (Ayton & Fischer, 2004; Burns, 2003; Estes, 1964; Lopes & Oden, 1987; Taylor & Brown, 1988). Indeed, there are many studies showing that beliefs in the causal process underlying the production of outcomes are the factors separating the gambler's fallacy from the hot hand (Ayton & Fischer, 2004; Burns & Corpus, 2004; Caruso, Waytz, & Epley, 2010). This suggests players have an awareness of the underlying processes and are reasoning about these problems but erroneously applying their everyday knowledge. Upon being reminded of the independence between events, players have been found to reduce endorsement of erroneous beliefs and reduce motivation to play (Benhsain, Taillefer, & Ladouceur, 2004). A model-based approach may explain findings that neither the normative or heuristics theories can.

In this chapter, I present data from two new studies that take a different approach to understanding how individuals conceive of gambles. In the first study, data from casino electronic roulette machines is used to study how agents process evidence (or do not process evidence) in a real-world game with their own money at stake. In the second study, the appearance of a gamble is experimentally manipulated to test how decisions on identical gambles depend on subjective perceptions. Ultimately, this chapter aims to motivate the case for a model-based account of decision making.

³ A thought to keep in mind, however, when considering judgments of randomness, is that the nature of the process of comparing a hypothesis of random structure to any other structure is that any evidence observed will always provide strong support in favour of the alternative hypothesis while no evidence can provide strong support in favour of randomness (Williams & T. L. Griffiths, 2008). Finding meaning and structure in randomness is not a mere bias but a difficult problem.

Study 1

A Role for Prior Beliefs in Roulette

Chance-based gambles are the paradigmatic tool used by researchers to evaluate the rationality of our decisions. These problems are simple to understand and often require only one easily-accessible physical process to generate outcomes. The spinning of a wheel of “fortune” is one classic example (Coombs, Bezembinder, & Goode, 1967). A wheel of (mis)fortune with unfavourable odds is ideal for study because the rational decision is clear: do not play. Roulette is one such unfavourable game.

Roulette is a popular table game at casinos that is based on the spinning of a ball on a wheel with numbered areas of equal size. Punters place bets on which area of the wheel the ball will land in, with both the wheel layout and the betting table layout incorporating randomness and patterns such as the placement of the numbers along the wheel and along the table and the alternation of black and red and of even and odd.

Despite the randomness built in to the outcome-generating process, strategies for beating the game abound. Thousands of websites offer tips and systems that guarantee profits at the table games in the casinos and at the gambling sites on the Internet. (While this researcher has not personally paid for access to learn these guaranteed-win systems, she is certain that the purveyors earn a greater profit from online sales than they do from employing the strategies themselves at a casino.) Many players believe they can beat this chance-based game despite its negative expected value. How can researchers studying gambling understand this divergence from optimality?

Looking for the gambler’s fallacy

In previous work, a common finding from investigations of wagering on chance-based gambles is unwarranted inferences from small samples of randomly-generated data, such as the gambler’s fallacy (a belief that the probability of an event is lowered immediately after it occurs) and the hot hand (a belief that the probability of an event is raised immediately after it occurs), which are in turn commonly interpreted in light of heuristics such as representativeness (a small sample of outcomes is used to infer properties of the underlying distribution of the population). Examples of this approach include Kahneman and Tversky (1971), Bar-Hillel and Wagenaar (1991), Rabin (2002), Croson and Sundali

(2005), and Barron and Leider (2010). While these studies have greatly advanced our understanding of behaviour, they share a common limitation of examining wagers conditional on previous outcomes.

For example, classification of a gambler as believing in the gambler's fallacy based on wagers alone is difficult: there is the unsolvable question of at what level to look for correlation to previous outcomes. Straight-up bets on the precise number that previously occurred? A different quadrant of the wheel than the previous outcome had landed in? A different area of the table layout? When an average of ten bets are being placed per person per spin (Croson & Sundali, 2005), it is difficult to assess reliable and meaningful dependencies between spins.

Secondly, the approach neglects prior beliefs and individual understanding of the problem of uncertainty. Assuming equivalent prior beliefs and interpretations of the problem is naive in other areas of research and such an assumption should be considered invalid here, too. The approach is akin to believing that all roulette punters come to the table with similar beliefs and simply testing whether they all weight incoming evidence in the same manner. The present researcher's interviews with casino customers indicated that punters held different strong beliefs about the game and used different strategies and focused on different features of the outcome and the environment.

To overcome this research approach limitation, the present study examines roulette wagering data to assess whether all roulette punters use data-based strategies. An alternative strategy based on priors will also be considered during classification. It is hypothesised that not all participants will exhibit data-based wagering, in support of a model-based account of wagering that incorporates prior beliefs.

Overview of dataset

Electronic roulette machine data were collected from a casino in central London that offers a range of table games, including black jack, poker, punto banco, roulette, dice, and video gaming machines. The data were recorded manually during non-peak hours of operation over several days with the permission and aid of the casino management. Wager and outcome information only were recorded; no private or identifying information about the customers was included. As with Croson and Sundali (2005) and

other field studies, this work also benefits from using field data. Participants in such studies stake their own money voluntarily and thus are genuinely incentivised to learn the game and make optimal decisions. Additionally, many participants have extensive experience with the game of roulette, unlike typical laboratory participants.

In the game of roulette, a win is determined by the resting position of a ball spun on a wheel. The wheel is divided into equal-size areas labelled with 37 numbers, giving each area (number) an equal probability of winning. At each spin of the wheel, any number occurs with a probability of 1 in 37 or 0.027. The bets offered at a roulette table range from single numbers (e.g., 24) to portions of the wheel (e.g., 3, 26, 0, 32, and 15) or portions of the table and number line (e.g., 25, 26, 27, 28, 29, 30) that have known and calculable odds of winning.

Players must place their bets on the table (or the video machine screen) during the allowed period before the spinning wheel begins to slow down. Players may place any and as many bets as they wish. The minimum bet size is £1.00 at the roulette tables of the casino in this study and £0.50 at machines.

Method

Participants

Participants of this study were customers of a London casino. During piloting, customers were approached and asked to participate in a psychology study and many declined (due to environmental circumstances, the precise number of participation request rejections was not recorded but is estimated to be 75% of those approached). Due to this challenge in interviewing customers, the design of this study was modified to analyse machine data only. During the later phases of the study, no customers were approached; data were collected after customers had left the gaming area. The data collected during these separate phases were combined for this analysis.

Of the study's 32 participants, 2 participants were removed from analysis because the stakes played or the profit won were significant outliers (± 2 SD from the mean), leaving 30 participants in the sample including 6 pilot participants. Only customers playing on electronic gaming machines were targeted; no further actions to address the possible

differences between punters who play at live table games and those who play at machines were taken.

Procedure

If customers were approached, they were approached only as they exited the gaming area. The researcher introduced herself and asked if they would anonymously answer questions about their gaming experience for a University College London research study. If the customer agreed, the researcher attempted to conduct a structured interview including questions about the customer's memory for their previous gaming session's wins and duration. Due to environmental circumstances (e.g., lack of privacy) and the sensitivity of the information being sought, several interviews strayed from the structured format.

During non-peak hours, the casino management granted the researcher access to the electronic roulette machines. Because no identifying information was available at the time of collection, the beginning and end of gaming sessions were deduced from wager behaviour. For example, long intervals between wagers, different wagering patterns, different wager sizes, and insertion of money after a nil balance were used as cues to indicate the start of a new session. A session was included in the dataset if it comprised more than 3 spins (minimum required for analysis of wagering patterns) but fewer than 50 spins (to reduce data processing). Relevant data per spin were manually recorded, including all bets, the outcome, the value of the punter's bank, and the time.

Results and discussion

Data on wagers and outcomes were recorded from electronic roulette machines to investigate strategy use by real-world players in chance-based games with high stakes. These data were analysed for wagering patterns within each gaming session and summarised across subjects.

The mean number of spins played during a single gaming session was 13 (SE = 1.71). The mean amount of money staked by the participants excluding any winnings used to continue play was £35.65 (SE = 8.22). The mean take at the end of each gaming session was -£8.38 (SE = 9.27). Of the 30 participants in the sample, 7 ended the session in

profit ($M = \pounds47.00$, $SE = 20.78$), 4 broke even and left the session with their original stake amount, and 19 lost money from the session overall ($M = -\pounds30.55$, $SE = 8.63$).

Wagering patterns

Participants' strategies were classified according to wagering patterns. Because different strategies may manifest in wagering patterns that appear similar and similar strategies may manifest in wagering patterns that appear different, precise classification according to known heuristics such as the gambler's fallacy or win-stay/lose-switch could not be completed. Rather, patterns in evidence processing were used for the classification. The first wager of a session represented the individual's prior beliefs. If more than 75% of bets for the majority of games were the same as the bets on the first game, the individual was classified as using a prior-based strategy. If bets were inconsistent, the wagers were compared to the observed outcomes. If any data-dependencies were found (e.g., changing bets to or changing amount staked on the previous game's outcome), the individual was classified as using a data-based strategy. If bets were inconsistent but did not appear to follow outcomes, the individual was classified as "other". Also explained below is an additional category that emerged from the analysis as independent of the model-based framework of evidence processing. These patterns are described below in Table 1.

	Prior-based n = 13	Data-based n = 7	Win-frequency n = 6	Other n = 4
Number of Plays	12.31 (2.93)	12.00 (3.34)	15.67 (4.50)	11.75 (2.50)
Amount Staked	£21.16 (4.71)	£61.65 (26.52)	£49.67 (23.30)	£23.75 (5.54)
Net Take	£-8.65 (5.52)	£-17.14 (34.31)	£10.33 (23.87)	£-20.25 (4.09)

Table 1. Summary of wagering behaviour across three different strategies (and other). Individuals' performances were averaged without weighting the number of plays. Standard deviations are shown in parentheses.

Prior-based wagering. These wagers exhibited patterns that indicated prior beliefs about the game were used for wagering that were independent of the incoming evidence. Forty-three percent of the sample was classified as relying heavily on prior beliefs for wagers. For example, one participant placed only straight-up bets on 0 for 12 consecutive spins. No spin during the session resulted in a zero. This pattern of wagering indicates the

participant brought a systematic belief to the table that withstood 11 items of (possibly) contradicting evidence. Betting on the same numbers repeatedly expresses a belief that, regardless of what the data suggest, the outcome-generating process is due to produce those numbers.

Data-based wagering. These wagers exhibited patterns that indicated the participant was highly sensitive to previous outcomes but not sensitive to prior beliefs as assessed by initial wagers. Although this classification is vague, it is believed that 20% of the sample fits this description. For example, one participant observed several times during his session that a number or two numbers positioned side-by-side on the table were repeated in short succession; he then placed a bet on those numbers, after having not previously placed bets on those numbers nor significantly increasing the number of bets being placed on the table generally. This pattern of bets suggests that the participant believed he could exploit observed dependencies in the outcome-generating process.

While we cannot infer the precise beliefs maintained by the participants in these two classifications about their respective roulette games, the different wagering and evidence processing methods used indicate that the beliefs about the likelihood of future outcomes—and therefore the underlying beliefs about the problem environment and the outcome-generating process—differ. In other words, the wagers differ between groups because the participants understand the problem of the roulette wheel in different ways.

Win frequency-maximisation wagering. An additional group emerged from the study that had not been hypothesised but was clear from the data. Twenty-three percent of the sample took on a strategy to maximise win frequency, covering the majority of the table with bets but without regard to prior beliefs or incoming evidence. These punters do not appear to be aiming for the conventional goal of maximizing profit. Instead, by betting on the majority of outcomes possible, these punters appear to aim to ensure a winning bet even if the high stakes paid reduce or even negate the winnings collected. However, simply ensuring a winning bet is insufficient. If this characterization were true, then we might expect to see many punters covering both sides of an outside bet, such as red and black, with the table minimum. Instead, these punters typically place straight-up, split, and square bets (see Figure 1 for an example), much like other punters, but in large quantities while risking no coverage on other numbers. This pattern indicates the punter

does not merely wish to pass the time playing the game for as long as possible but instead strategises to experience large wins regardless of the cost. While this pattern indicates a degree of reliance on prior beliefs or incoming data for selecting which numbers to cover, the overwhelming emphasis is on win frequency-maximisation.

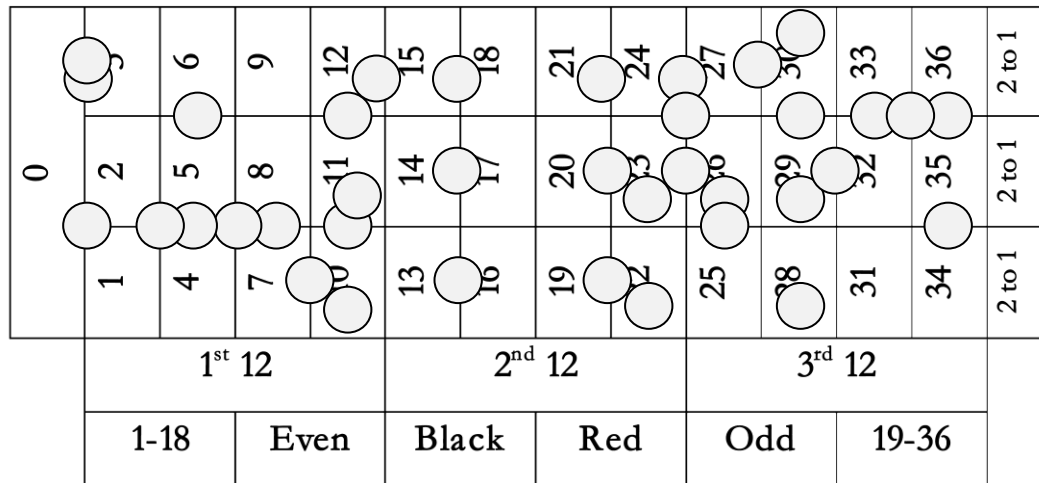


Figure 1. An example of a participant's win-frequency maximisation strategy. Each circle represents a bet; where the circle covers more than one number, the amount wagered is split evenly. Across the 37 possible outcomes, the numbers 9 and 31 are not covered by any bets but all other numbers have from £0.75 to £11.50 bets. A total of £214.00 was wagered. In instances where this strategy is implemented but the total wager is lower, the amount per bet decreases, e.g., to a range of £0.50 to £1.50 bets.

This descriptive account of real-world wagering demonstrates that there are clear divisions in strategy used by decision makers faced with straightforward chance-based gambles. Punters examined in this dataset deviated from randomness in at least two systematic ways: relying on prior beliefs and strategies despite incoming evidence *or* relying on incoming evidence with no clear dependencies on prior beliefs. Contrary to the assumptions of much of previous work, the majority of players in the dataset appeared to place bets independent of the incoming evidence. Indeed, to believe otherwise would amount to expecting everyone at a table to bet largely the same way. This division also indicates a deep difference in how the participants perceive the underlying outcome-generating process that they are wagering on. Those punters whose wagers reflect their prior beliefs may believe they can control the outcome or that they have a knowledge edge over the house; they may be willing to eat losses without changing their strategy because they understand randomness affects outcomes and the predicted payoff is still yet to come. Those punters whose wagers reflect the incoming evidence may believe that they can match the observed probabilities to exploit biases in

the outcome-generating process. These are just two interpretations that capture the important differences in the wagering patterns found here but there are as many strategies as there are players. This analysis shows that researchers studying gambling must consider individual beliefs even for problems as straightforward as roulette.

Study 2

More Than One Way to Throw the Dice

As was shown in the previous chapter on roulette play in a casino, people maintain strikingly different beliefs despite years of feedback and knowledge of a well-known game's randomising process. In this study, I aim to revisit one of the commonly cited causes for this behaviour - the illusion of control bias.

An illusion of skill

The research into the illusion of control bias suggests that the inclusion of skill-related factors in a game induces a skill orientation and consequently an illusion of control, despite the game being uncontrollable. There are a number of factors that have been shown to induce this illusion: choice, stimulus or response familiarity, passive or active involvement, competition (E. J. Langer, 1975); more frequent successes early on in play (E. J. Langer & Roth, 1975); more frequent successes at any time (Alloy & Abramson, 1979).

Studies investigating the decision makers' thought processes may explain these results. There is some evidence to support an account of decision making in chance-based gambling that goes beyond the resemblance to skill-based games and considers the perceptions of the decision makers. Analysis of verbalizations made by slot machine players finds a correlation between the amount of money staked and the number of thoughts spontaneously verbalised, with the majority of thoughts being false statements about control over outcomes (Delfabbro & Winefeld, 2000). This study suggests that players perceive a representation about how the slot machine outcomes are produced that does not match reality.

However, subtle interventions have been shown to reveal an awareness of reality. A study of the thoughts expressed by regular gamblers shows that participants' risk-taking behaviour was dependent on the beliefs expressed by an accomplice: statements about luck and skill induced more play while statements about chance (e.g., "it's too bad that we don't control chance") induced relatively less play (Caron & Ladouceur, 2003). And Martinez, Le Floch, Gaffié, and Villejoubert (in press) demonstrate that observing another player's successes leads to increased belief in control over the game but when the

other players' successes are attributed to luck, the increase is eliminated. These studies illustrate that individuals seem able to rationally use information about games. And, the illusion may be eliminated in multi-shot gambles (J. J. Koehler, Gibbs, & Hogarth, 1994), when playing with high stakes (Dunn & Wilson, 1990), and when experiencing early losses (E. J. Langer & Roth, 1975). Resemblance to skill cues is insufficient for explaining the bias.

A model of skill

A model-based account, in contrast, examines the agent's underlying belief about the game and predicts that it is the agent's model of the outcome-generating process and not simply the appearance of skill per se, that induces willingness to gamble. If the agent believes he has at least some control over the outcome—through skill cues, most likely—he may play. These predictions are aligned with the original illusion of control hypothesis.

However, increasing skill perceptions may also induce a decreased willingness to gamble. By examining the agent's underlying model or beliefs about the uncertainty of the gamble, it becomes evident that the relationship can be non-linear. If the agent perceives that the outcome-generating process is exceedingly complex and impenetrable to his control—again, through skill cues, most likely—he may not play. In contrast to the original illusion of control hypothesis that increased resemblance to a skill-based game leads to an increased willingness to gamble, a model-based account hypothesises that willingness to gamble decreases as the perceived complexity of the outcome-generating process exceeds perceived control. This is similar to the control heuristic account posited by Thompson, Armstrong and Thomas (1998), whereby the bias relies on perception of intentionality and perception of connection between action and outcome. These two ideas share a concept of subjective mental representations of the outcome-generating process.

The present study compares the model-based account as described above to the traditional skill-based account, which predicts that perceived control and willingness to gamble follow a linear relationship, whereby increased perceived control induces increased willingness to participate in chance-based gambles.

For our study of decision making in chance-based gambling we must first make an assumption: the agent's choice to participate in a negative expected value gamble is assumed to indicate his belief that there is unknown uncertainty in the outcome-generating process. That is to say, the agent who chooses to play believes there is more to the game than just randomness. We have seen positive evidence for this assumption earlier in this chapter in the analysis and discussion of casino roulette wagers. Although participants of games such as these or national lotteries may claim to play for social and personal benefits, many employ strategies and maintain beliefs that suggest they perceive some amount of control over outcomes (Bar-Hillel & Neter, 1996). We believe this is a reasonable assumption. For those agents who play exclusively for social and personal benefits, an entirely different model comprising excitement and passage-of-time may apply to their perspective of the game; we are not concerned with these non-economic factors in this discussion.

Given this assumption, choosing to play a negative expected value gamble demonstrates a belief of some degree of control over the gamble's outcome. A punter who repeatedly puts down chips on the roulette table believes it is possible to gain an edge over the house and make at least a marginal profit. In previous research, choosing to play a negative expected value game has been used to assess willingness to gamble, as in E. J. Langer (1975).

Overview of experiment

Participants attempted to predict the outcome of a die throw as many times as they wished. For all participants, the game's outcome is ultimately determined by the throw of a single die and has the same negative expected value regardless of experimental setup; in formal terms, each participant was offered the same gamble on the outcome of the throw of a single die. However, it was predicted that participants' willingness to gamble would be better predicted by their perceived control over the game's outcome rather than the game's resemblance to skill games.

The game cost 10 pence to play and paid at 4:1 odds; an example explained that upon correctly predicting the outcome, the participant would collect 50 pence, including the original stake of 10 pence, for that die throw. The game continued until the participant chose to quit. All participants received the same instructions. While not stated explicitly

in the instructions, the gamble offered was based on chance and had a significant house edge, because the outcome was ultimately determined by the throw of a die and actual winning odds were 5:1 but payout odds were set at 4:1.

Method

Design and materials

The experiment tested the relationship between the perceived control required to complete the task and willingness to gamble. The perceived control required to complete the task was manipulated by increasing the complexity of the choice set. The design comprised a single between-groups factor with three levels of perceived control required to complete the task: no control, attainable control, and unattainable control. Participants were randomly assigned to an experimental condition and remained unaware of any alternative task specifications. In each condition, participants were informed that their compensation would depend on the outcomes of the decisions and choices they made in the task.

All dice used in the task were fair six-sided dice of identical size varying only in colour. One group of participants was presented with six dice comprising three green dice and three blue dice. From these dice, participants could choose one die to be thrown by the experimenter (this choice was repeated, with replacement, for each die throw of the task). A second group of participants was presented with the same six dice but were unable to choose which die to throw; the die was drawn blindly by the participant from an opaque sack out of the six possible dice. The first group, with a choice of which die to throw, is labelled the “attainable control” group and the second group, with no choice, is labelled the “no control” group. A third group of participants was presented with 50 dice, including the 6 dice seen by the other groups and also additional dice of orange, red, black, and white colours (5 different colours). This group, with the choice of which die to throw from a complex choice set, is labelled the “unattainable control” group.

For comparison to previous literature, these experimental groups may be reinterpreted in the language of the illusion of control hypothesis: “no control” (blind draw of 1 out of 6 dice), “low degree of control” (choice of 1 out of 6 dice), and “high degree of control” (choice of 1 out of 50 dice).

Participants

Thirty-five participants were recruited to participate in a paid study on gambling from the University College London Psychology Department Subject Pool, Gumtree (a popular online UK notice board), and local newspapers. Outliers based on number of die throws played (± 2 SD) were removed; this resulted in one participant, who chose to play for 40 throws, being removed. Participants were randomly assigned to conditions, resulting in a final sample of 12 participants in the group designed to perceive no control, 11 perceived attainable control, and 11 perceived unattainable control.

Inventories were administered after the task to measure numeracy and gambling problem severity (see Appendices 1 and 2). This sample of participants scored 86.12% (SD = 3.44%) on the numeracy scale, where 100.00% reflects a perfect score, and 2.85 (SD = 0.63) on the problem gambling severity index, where a score above 2.00 reflects potential gambling problems.

Procedure

This experiment was run in conjunction with three unrelated experimental tasks on gambling. The tasks were administered in the same order for all participants; this experiment was run second in the series, following a task for which participants viewed videos of horse races and made wagers on which horse they believed would win. It is assumed there are no carryover effects between experimental tasks.

At the start of this task, participants were seated at a table with instructions and dice, which were laid out according to the specifications of the randomly assigned condition. The instructions, which were the same for all participants regardless of experimental condition, described the game, the cost to play, the payout odds, and an example outcome. The instructions also explained that participants were not obliged to play the task and could choose to pass without penalty; this was emphasised orally by the experimenter because pilot participants indicated they felt an obligation to participate.

If a participant chose to not gamble, the dice and instructions were removed and the next task was administered. If a participant chose to gamble, the task followed the specifications of the assigned condition. After each die throw, the experimenter recorded the outcome and offered another die throw at the same cost and payout odds. If the

experimental condition required drawing or choosing a die, the draw or choice was repeated for each throw, with replacement. The task continued as long as the participant wished to play.

Results

In this experiment, a between-subjects design compared the willingness to gamble of participants in each of three groups, measured by the number of times the participants chose to play the dice game. Critically, the game presented to each group was the same: to predict the outcome of a single die throw. The three groups' games were varied only on how participants conceived of their control over the outcome-generating process. If the participant chose to play the game, the number of times the participant chose to play was recorded; otherwise, the measure was recorded as zero.

The mean number of times participants chose to play overall across the three conditions was 3.88 (SE = 0.71). The mean number of wins by individuals was 0.68 (SE = 0.20). Group means are shown in Figure 2.

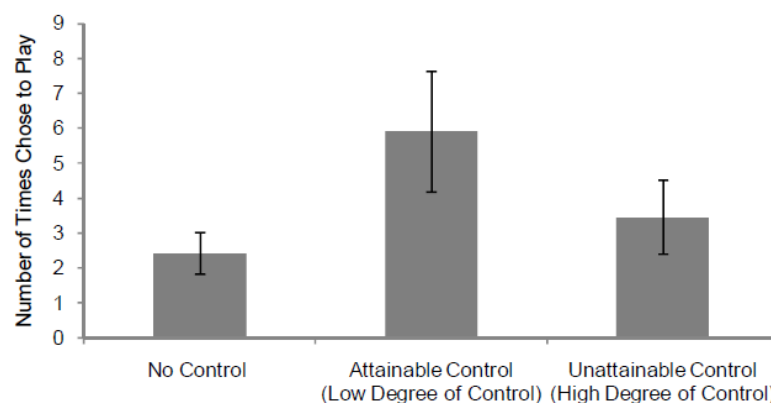


Figure 2. Mean number of plays (and standard error bars) for each experimental condition.

Overall, the model-based account predicts a non-linear relationship between perceived control as manipulated in the experiment and willingness to gamble, in contrast to the illusion of control hypothesis that predicts an increasing linear relationship. Orthogonal contrasts testing the relationship between perceived control and willingness to gamble across the three groups support the model-based account: the group that was designed to perceive full control over the outcome gambled more frequently than the other groups combined (Beta = 0.34, $p = 0.05$); the linear contrast testing the illusion of control

hypothesis was not supported (Beta = 0.10, ns). However, the overall model did not reach significance ($F(2,31) = 2.28, p = 0.12$).

The addition of a covariate measuring the number of correct winning predictions significantly increased explanatory power (Beta = 0.67, $p < 0.001$; R-square change = 0.42, F change (1,30) = 29.35, $p < 0.01$) and the predicted non-linear relationship remained significant. The addition of parameters for numeracy and gambling problem severity did not significantly improve the model.

To analyse the effect of early wins on willingness to continue gambling, participants who had correctly predicted at least one die throw within the first three throws (13 participants) were compared to those who had chosen to play but not won a die throw in the same period (11 participants). Groups were no different from each other in terms of mean gambles taken (early wins: $M = 5.55, SE = 1.50$; no early wins: $M = 5.46, SE = 0.83$; $F(1,23) = 0.003, ns$).

Discussion

This experiment manipulated participants' conceptions of uncertainty by varying only the framing of a chance-based gamble to understand the relationship between the task's resemblance to skill and willingness to gamble. Despite each group of participants facing the same gamble on a single die throw, manipulations of perceived control created significant differences.

While the gamble offered to each group of participants was formally identical, the framing was varied in an ordinal manner that meaningfully overlapped with perceived control: a game that offered no control equated to having no control over the outcome-generating process; a game that offered a few choices and some degree of control equated to requiring attainable control over the outcome; and a game that offered many choices and a high degree of control equated to requiring an unattainable level of control over the outcome. As hypothesised, a model-based account of decision making predicted participants' willingness to gamble better than resemblance to a skill-based gamble.

By simply increasing the number of choices available and therefore complicating the participants' models of the underlying outcome-generating process, we found a non-

linear relationship with perception of skill required and willingness to gamble. Participant willingness to play was not dependent on the simple absolute number of options and semblance to skill. It appears that the framing of the dice influenced participant understanding of the process by which outcomes were produced. Even in an environment with clear and simple rules such as a die throw, individuals' perceptions of the underlying process are critical in understanding behaviour.

The number of successful wins during play also had a significant correlation with the participants' willingness to gamble. Although the causal direction of the relationship remains uncertain, the absence of a significant increase in number of games played due to early wins suggests that any illusion of control or mastery over the game is not a factor in willingness to gamble. Rather, it seems that willingness to gamble is largely dependent on the gamble offered.

Although this study was completed with real dice and real monetary consequences, participants in this laboratory experiment did not face the prospect of overall losses. Some participants expressed a feeling of obligation to play as part of the experiment while some others played for fun rather than to maximise payments. However, it is not expected that these participants affected the validity of the findings presented here as participants were randomly assigned to conditions.

Further research with this paradigm should increase the sample size used to increase power and reach a significant relationship between the groups. Also, manipulating larger and smaller perceived differences between the framings of the dice may produce interesting results for identifying boundary conditions.

In summary, this result provides evidence in favour of a theory of decision making in gambling that is grounded in subjective models of the underlying uncertainty. The artificial differences in game framing led participants to perceive the game and their chances of winning differently. In support of the previous literature discussed, decision makers appear to use environmental cues (dice framing) and outcomes (wins and losses) to develop beliefs about gambles that may be altogether unrelated from the gambles' true underlying processes. The effects of such mental representations in even a clearly

chance-based environment indicates that studying optimal decision making under more complicated circumstances may be more challenging than normative models assume.

Chapter 3

Fixed and Unknown Uncertainty

Consider now a second coin. This coin is different from the one considered at the start of the previous chapter: it is slightly heavier on one side. The decision making agent is aware that this coin is systematically biased but does not know the precise amount, unlike in the case of a fair coin for which the bias is zero. Although the formal problem presented in a gamble based on a coin of unknown bias is solvable⁴, the problem is immediately made more complex than the known bias version.

Gambles of unknown but fixed uncertainty are effectively inference problems in which learning about hidden underlying outcome distributions and outcome-generating processes requires generalising from samples. The agent's learning over an accumulation of evidence—including how evidence is collected, processed, and retrieved from memory—becomes critical for judgment and decision making. There are many factors that must be evaluated and integrated: the knowledge that the agent brings to the problem, the incomplete description of the environment provided, possibly limited sampling of the action-outcome space, and hypotheses and beliefs conditional on these outcomes.

Beliefs about future events

Before taking a gamble, a decision making agent, it is assumed, considers whether he will win money and rejects unfavourable gambles. Most would agree that the likelihood of one's lottery ticket winning is next to nil. But people who purchase lottery tickets must believe there is at least some likelihood that their ticket might win. Intuitively, this belief is a matter of degree – I am *more likely* to be struck by lightning than to win the lottery. What does the agent have in mind when purchasing a lottery ticket?

Up to this point, probabilities and beliefs about probabilities have not yet factored into this discussion of decision making in gambling. As the content matter shifts to examining the cognitive processes underlying the behaviour observed in Chapter 2, probabilities

⁴ However, the decision making agent may never know that he has solved the problem and converged on the correct bias value.

become more relevant. It is a topic of debate whether it is appropriate to use a probabilistic approach when studying human decision making (Harman, 1986; Kyburg, 1961; Oaksford & Chater, 2007). In the lottery example, however, it seems clear that decision makers do consider future uncertain events with degrees of belief. The probabilistic approach holds in the weak sense, whereby decision makers understand some events are more likely than others, but also in the strong form, whereby people use probabilities to represent future events and estimate expected value. For example, using the odds information from a bookmaker requires at least an understanding of probabilities. For the remainder of the thesis, I put aside the issue of whether the probabilistic approach is appropriate. The arguments for and against stray into the philosophical realm and, for my intentions, confuse the issue at hand.

Instead, I will focus on how gamblers use probabilities—when aggregated from experience—in their decision making. The following studies will explore how decision makers represent and utilise probabilities. We will be interested in not only the judgments and decisions that agents make but also how the agent arrives at that decision.

A primer on slot machines

Hidden behind the interfaces of video lottery terminals, poker machines, fruit machines, slot machines, and one-armed bandits are the mechanisms that determine whether the outcome of a play is a win or a loss. Much like dice and roulette games, slot machines are based on chance, with odds stacked in the house's favour. However, unlike dice and roulette games, the precise odds and how they are calculated are not transparent. While a die has six sides each with equal likelihood of landing face-up, a slot machine has an unknown number of symbols in unknown locations and ratios on each reel and an unknown algorithm determining the outcomes. In brief, a slot machine is an extremely difficult type of inductive inference task. And millions of punters on the Internet and in casinos and pubs try daily to solve it without success.

At most machines today, the only information known to players is a long-run payout percentage and a succinct payout table that lists which outcomes are associated with which combinations of symbols (or, as they might have you believe: which combinations of symbols cause which outcomes). Random number generators and proprietary algorithms provide the uncertainty element of the game. With hidden outcome-

generating processes and incomplete information about the outcome distribution, unknown uncertainty is both the appeal and the curse of these machines, as it is difficult to predict future outcomes and understand the consequences of one's actions. Because it is true that some outcomes are more or less probable than others, with particularly desirable outcomes being particularly less probable, it is not difficult to see how a player might mistakenly interpret the underlying outcome-generating process to include skill. Indeed, among some gamblers, successful play is considered a skill; M. D. Griffiths (1990), in a study analysing the verbalisations of slot machine players, found that players attributed wins to experience and skill.

In this chapter, I focus on how decision makers learn about hidden probability distributions and processes under unknown uncertainty. I present data from two new experimental studies and a modelling simulation. All experimental studies use a computer simulation of a slot machine (also known as a fruit machine), which has a fixed random outcome-generating process but hidden outcome probability distribution. The first study examined retrospective judgments based on small samples to understand how memory and experience feed into inference about the future. The second study manipulated participants' prior knowledge about the outcome space to study the online updating of hypotheses in a task to learn an underlying probability distribution. A modelling simulation further explores how hypothesis evaluation contributes to the shaping of subjective probabilities. These experiments aim to examine the processes underlying subjective beliefs and repeated decision making over time.

Study 3

Remembering to Lose from Slot Machines

Memory is a major part of our judgment and decision making but is also one of the most malleable and dynamic processes in cognition. Appreciating how it contributes to beliefs about gambles is critical for advancing this discussion on gambling judgments and decisions. In this study, I present an experimental task with both objective and hedonic value to understand how decision makers process evidence for subsequent judgments and decisions.

Through interviews with slot machine players and observation and cataloguing of their verbalisations during play, several conclusions about how gamblers think about the slot machines have emerged: punters play “with” money rather than “for” money; punters’ primary goal is to make their money last for as long as possible rather than to gain as much as possible; and punters are aware of the negative expected value and chance elements of the games they play (Delfabbro, 2004). Studies of arousal confirm that regular gamblers have increased heart rates during and after play and particularly after wins, near-misses, and specialist play characteristics such as nudge and bonus features (Moodie & Finnigan, 2005). Research using clinical and physiological data supports this idea that monetary losses are the price paid for the excitement of gambling (Bruce & J. E. V. Johnson, 1995; Gilovich & Douglas, 1986; M. D. Griffiths, 1993). Slot machine play, it would seem, is about the experience.

The peak-end rule

An essential distinction here is between *instant* and *remembered* utility. While instant utility is the pleasure or pain we feel at a moment—a fleeting feeling that is replaced by the next moment’s instant utility—remembered utility is the global assessment we make retrospectively that encompasses those moments and becomes our memory for the experience. While both are useful constructs in their own right, a conflict between the validity of a person’s total utility (sum of instant utilities) and remembered utility of an experience emerges when evaluating *temporally-extended outcomes*, or experiences that take place over time (Kahneman, Wakker, & Sarin, 1997).

Previous experimental research has explored the instant and remembered utilities of temporally-extended outcomes in a range of contexts, including film clips (Fredrickson & Kahneman, 1993), medical procedures (Redelmeier & Tversky, 1996) and sounds (Schreiber & Kahneman, 2000). In these experiments, researchers arrived at three main findings: the peak-end rule and its two related effects of duration neglect and violations of monotonicity.

The peak-end rule states that a simple average of the instant utilities (or disutilities) felt at the peak intensity of an experience and at the end of an experience determines the global assessment given retrospectively. For example, consider Redelmeier and Kahneman (1996), a study of patients' retrospective evaluations of colonoscopies. Patients experienced the same routine procedure, but doctors prolonged the experience for some, adding additional pain at a lower intensity. Although the patients in the shorter procedure group experienced objectively less pain, they rated their experience more negatively than the patients in the longer procedure group. Patients' memories for the pain felt during the colonoscopies matched a peak-end average better than an aggregation of total disutility.

While previous research has provided strong evidence for the peak-end rule and catalogued its factors and behaviour to some depth (see Schreiber and Kahneman (2000)), there still remain questions about its determinants and applicability. The research field to this point has focused on passively experienced and primarily sensory stimuli to evoke hedonic responses, such as colonoscopies and aversive sounds. But there is an interesting and still novel area of research in the interaction between hedonic and monetary utility in gambling.

Sequences of monetary gains or losses are common experiences in the real world. Regular bills and payments are ubiquitous and the ability to make accurate retrospective judgments about them may have a serious impact on our decision making. Economic theory would have our judgments of monetary sequences follow normative rules, such as the average or total amount of money gained or lost; rational people prefer more money to less. T. Langer, Sarin, and Weber (2005) tested this concept and observed that, rather than use normative rules, people overweighed peak-end values. However, the result was not found in tasks of little affective experience; the effect was strongest on performance-

based tasks. The extension to monetary sequences is still unclear. Gambling outcomes are yet more ambiguous: gambling has acutely hedonic wins and losses but also cognitive biases such as illusion of skill, tendency to scrutinise losses, or inclination to transform losses into “near-misses” (Gilovich, 1983; Reid, 1986) that modify the hedonic profile of a temporally-extended outcome away from its monetary values.

Estimation of monetary sequences

The objective nature of monetary stimuli broadens the scope of possible analyses beyond that of previous research. In addition to peak-end choice evaluations comparing sequences, these experiments can also examine numerical estimation, a concept critical to gambling judgments and decisions.

Most of the numerical estimation literature focuses on numerosity or magnitude, largely neglecting the problem of arithmetic involving sequences. Results from T. Langer, et al. (2005) suggest that subjects keep running totals, but this was found for sequences of 10 or fewer numbers under the explicit direction to attend to the sequences. It is still unknown how subjects evaluate longer trials, which resemble real world scenarios that persist beyond 10 instances (a session at a fruit machine might continue beyond 100 spins).

Results from this study may also inform current debates on how sequences of evidence are encoded in memory. If estimations are reasonably accurate despite extremely high peak and end values, the findings may support the notion of an eager learner who updates beliefs in an online manner. In contrast, if the peak-end hypothesis is supported, the findings may suggest a lazy memory-based learner whose judgments are affected by availability and saliency of data in memory.

Overview of experiment

To study how decision making agents generalise from small samples and make judgments about their environment, a within-subjects design compared retrospective judgments about slot machines after short sessions of play. Machines used different outcome distributions from which to generate payouts. In particular, a machine with isolated extremely high values was pitted against one that was free of such extremely high values

but paid more in total. Normative reward maximizing behaviour (e.g., using a rule such as maximising total session payout) predicts a preference for any sequence with a higher overall reward regardless of the features of the sequence. In contrast, a heuristic account, such as the peak-end rule, predicts that extremely salient positive “peak” moments of the experience bias preferences and estimation judgments.

Participants played computer-simulated slot machines. The simulation appeared to function like an actual slot machine, though running on a conventional PC. Participants clicked a button to activate the spinning of the machine’s reels and several seconds later, when the reels came to a stop, the value of points awarded based on the combination of symbols was shown; there was no cost to play. The program then waited for the participant to play again. The machine did not show cumulative winnings.

The task included blocks of 2 sequences with 25 trials each. Within a block, participants played a sequence of 25 trials from a slot machine followed by a sequence of 25 trials from a different slot machine, before providing evaluations of the pair of machines, including total payout received and preferences. The task included several experimental manipulations but only the two most relevant to the aims of this paper will be presented here; the findings reported generalise to the study overall.

Method

Design and materials

A within-subjects design that manipulated the outcome distributions underlying slot machines was used. Judgments of total pay and preference for machine were collected. This study tests whether the presence of high peak values affects the accuracy of retrospective judgments of a sequence of outcomes compared to performance on a neutral sequence.

One sequence was designed to have high “peak” or “jackpot” values at the middle (serial position 12 out of 25; value of 50 pence) and end (serial position of 25; value of 25 pence), with the remaining 23 outcomes sampled from a limited outcome space to maintain a fixed level of total payout constant across all participants. This sequence was pitted against a neutral sequence, which comprised outcomes sampled from a limited outcome space with no extreme payouts to maintain a fixed level of total payout constant

across all participants that was higher than the paired peak sequence. It also tested whether participants preferred the high peak values, despite the peak sequence's lower overall pay; the skewed sequence paid 19.35% fewer points than the control sequence paired with it. Another block of sequences was administered to serve as a control measure of how participants evaluated a pair of sequences without any extreme values. The payout discriminability between sequences was controlled in all blocks and levels of accuracy in judgments of total payouts and proportions of the sample choosing the higher-paying sequence were used as standards of performance for the paradigm. Presentation of the two blocks was counterbalanced.

Participants

Seventeen participants were recruited via the UCL Psychology Subject Pool and told they would receive a fixed payment for their time and an opportunity for bonus pay dependent on performance in the experiment.

Procedure

Participants were told they would play virtual slot machines on the computer and afterwards answer questions about their experience. The computer program presented instructions for operating the slot machines and provided a payout table listing the combinations of symbols and their payouts. Participants were permitted as much time as needed to understand the task. During the fruit machine session, the participants controlled the speed at which play progressed, but the duration of presentation of payouts remained constant at 1.5 seconds. The end of the first sequence in a block of was followed by a short break of a blank screen before the start of the second sequence. After completing the second sequence of a block, the program prompted the participant to answer evaluation questions, one at a time, in a random order except with the preference question (at which machine they preferred to play an additional high stakes session) always appearing last.

Results

Percent error is used in these analyses rather than absolute points because the payout totals varied across sequences; errors measured by percentages enables a standard for comparison.

Estimations of total pay

In the control condition, participants were able to distinguish between the two sequences and correctly identify which was higher-paying (difference in total payout between sequences of 30 points; higher-paying: $M = 128.52$, $SE = 9.80$; lower-paying: $M = 96.53$, $SE = 9.05$; $F(1,16) = 38.47$, $p < .001$, $\eta_p^2 = 0.71$). Participants' estimates of total payouts of both sequences were significantly lower than actual sums (higher-paying sequence: $t(16) = 2.70$, $p = .02$; lower-paying sequence: $t(16) = 3.14$, $p = 0.01$) with an average underestimation error of 19.97% ($SE = 4.8\%$). The magnitude of these errors did not significantly vary between sequences ($F(1,16) = 2.29$, ns). These data are illustrated in Figure 3 on the left-side panel. This analysis provides confidence in people's ability to discriminate between higher- and lower-paying sequences and to make reasonably accurate judgments regardless of high or low absolute payout in this paradigm.

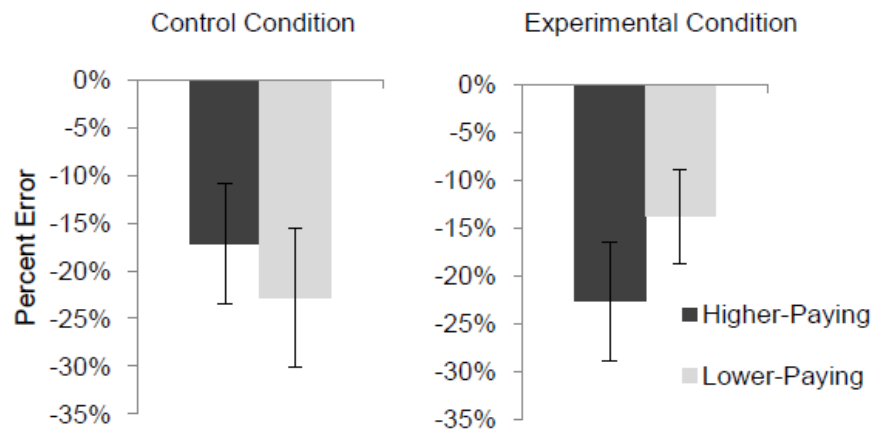


Figure 3. Percent error in mean participant estimates compared to observed sequence for each participant. Negative data points indicate participant underestimation. In the right-side panel, the grey bar represents the peak sequence. Error bars show standard errors of the means.

The same analyses were conducted for the pair of sequences that included a peak sequence with extremely high values. Unlike in the control condition, a repeated-measures ANOVA found that estimates of the sequence sums were not significantly different (difference in total payout between sequences of 30 points; higher-paying: $M = 123.94$, $SE = 9.83$; lower-paying: $M = 112.24$, $SE = 6.41$; $F(1,16) = 3.13$, ns). As seen in the control condition, the overall estimation of sequence payouts was significantly lower

than actual observed payouts (higher-paying: $t(16) = 3.67$, $p = 0.01$; lower-paying: $t(16) = 2.77$, $p = 0.01$). These data are seen in Figure 3 in the right-side panel.

To compare judgment performance between conditions with peak values and with neutral values, a repeated-measures ANOVA was used with factors of relative payment (high or low) and sequence type (peak or neutral). As found in the above analyses, the main effect of relative payment was not significant ($F(1,16) = 0.31$, ns). However, the interaction between sequence type and relative payment was significant ($F(1, 16) = 7.03$, $p = 0.02$, $\eta_p^2 = 0.31$), suggesting that participants evaluated the lower-paying sequence differently when it contained extremely high values.

Preferences

Preference data assessing participants' hedonic evaluations confirmed the informational estimation findings. In the control condition, all participants preferred the higher-paying session, as expected.

Choice data for the peak-end sequence were compared to the control condition. In evaluating the pair with a peak-end sequence, five participants chose to play again on the machine that paid the peak sequence of payouts, compared to no participants in the control condition. Nonparametric analysis found this to be a significant difference (one-tailed $p = 0.03$, McNemar test), showing that more participants mistakenly preferred the lower-paying session when it contained extremely high payout values.

Discussion

After observing paired sequences of slot machine outcomes, participants were asked to make retrospective evaluations. In each block of two sequences, one sequence paid relatively more overall. A control pair of sequences captured baseline performance in the task: participants successfully identified which machine of the pair was higher paying but committed an overall unbiased error of underestimation of total payout. When observing a pair of outcome sequences containing extremely high peak and end values, participants committed the same overall underestimation error but reported upwardly biased estimations of the sequence sum with high peak and end values. Furthermore, more participants preferred the lower-paying sequence when it had high peak and end values than when it did not.

Overall, participants were poor estimators of sequence sums. With an average underestimation of 19.97% in the baseline condition, participants made serious errors. Participants did not use the simple arithmetic strategies typically used in addition or estimation tasks; however, given that estimations for sums of compared sessions reflected actual differences between those sessions, it seems likely that participants did use a strategy of some kind.

These findings provide mixed support for the peak-end rule. As the rule predicted, participants' retrospective estimates of total payout were mistakenly higher for the peak-end sequence. Additionally, more participants preferred the machine with the extremely high values despite its lower payout total, also as the rule predicted. However, the positive evidence is weak. In contrast to the qualitative pattern found in previous studies whereby the lower-pain sequence with high peak and end values was judged as *more* painful than the baseline control sequence (Redelmeier & Kahneman, 1996), participants here still judged the lower-paying sequence as lower-paying. Additionally, the choice data also showed only a weak preference for the machine with the high values with the majority of participants still choosing the normatively correct higher-paying option.

The consequences of these findings are at this point limited but, with further research, may prove to be important. Actual slot machines typically display cumulative winnings on screen, along with other bank information such as credits. Because this experimental paradigm does not display cumulative winnings during play, the errors in retrospective estimations reported here may be irrelevant. However, in the long-term, once players have stepped away from the machine and no longer have such information accessible, the peak-end rule may affect subsequent retrospective estimations. Such biased judgments may feed into decisions of whether to begin playing again, as Redelmeier and Kahneman (1996) found that biased memories of colonoscopy procedures affected decisions to repeat the procedure. Indeed, gamblers have been noted to dismiss losses but remember wins (Gilovich, 1983); this tendency may interact with the errors in retrospective evaluation to escalate the size of the error over time. Future research should investigate the duration of the peak-end effect on retrospective evaluations of winnings.

While several of the results presented here deviate from expectations, the differences may be due to translation into a new paradigm. The peak-end rule had not yet been tested with gambling wins as in this paradigm, and the weaker findings may be caused by paradigmatic limitations. More extreme peak values and longer sequences may shift the results.

However, where the normative and heuristic theories fall short in explaining participant performance in the present study, an alternative account may be more fitting. Recall that participants viewed payout tables at the start of the experimental session that identified the range of outcomes possible, including the extremely high values. Neither the normative additive or heuristic accounts explicitly consider such environmental cues. If the participants enter the task with prior beliefs about the distribution of outcomes they are likely to view during the sequence and update these beliefs online as new evidence is presented, then retrospective underestimation in control conditions is likely to occur due to the absence of the extremely high values. A model-based account provides the necessary assumptions to develop this explanation of participant judgments, as does any Bayesian account of learning whereby beliefs (but not values, like a cumulative average, per se) are updated over time. A connection based on these data is tentative, and testing in Study 4 of this chapter will examine the idea further.

Study 4

The Diminishing Value of Expecting a Jackpot

From the preceding studies, it has been shown that decision making agents may approach the same problem with different models and that these differences result in a range of beliefs about the underlying outcome-generating process. The data have also shown that the cognitive processes involved in this belief updating cannot be simply explained by the rational and heuristic accounts of decision making that are conventionally applied to gambling behaviour. In the present study, the process of belief updating is examined further to understand how and why slot machine data suggest a model-based account of decision making.

Hypothesis testing

In gambling, focal hypotheses are a central part of any gambling decision. Decision makers may position thoughts as about a particular outcome and strategies as about their own likelihood of winning. If I roll the dice in this way, will I get a seven? A common finding in psychological research is the confirmation bias, whereby judgment and decision making are biased toward selected focal hypotheses and alternative hypotheses are underrepresented. As a consequence of this bias in information search, evidence that lowers the likelihood of the focal hypothesis being true or even disconfirms it may not be considered (Klayman & Ha, 1987). And as another consequence in hypothesis evaluation, people may believe data directly support a hypothesis when in fact it is not diagnostic or more strongly supports alternative hypotheses (Doherty, Mynatt, Tweney, & Schiavo, 1979; Fischhoff & Beyth-Marom, 1983; M. Snyder & Swann, 1978). In some instances, decision makers, in taking action based on their focal hypothesis, may come to believe the belief is true (rather than merely a hypothesis) and ignore the signals from evidence that might indicate otherwise (Gärdenfors & Sahlin, 1982). Nickerson (1998) provides an extremely thorough review of the confirmation bias and its applications and consequences.

To investigate hypothesis testing in betting judgments, Gibson, Sanbonmatsu, and Posavac (1997) asked participants to consider four basketball teams in a computer-

simulated tournament. Participants randomly selected a focal team using a spinner board and then reported their judgment on the likelihood of that team winning. In one condition, participants were prompted to simply judge the likelihood of winning. In a second condition, participants were prompted to provide the likelihood assessment and also explain how or why the team might win. A control group of participants simply gave their estimates for each of the four teams. Participants were then offered the opportunity to bet on which of the four teams would win, with all teams being given equal odds of winning. Analysis found that likelihood judgments in all conditions were greater than chance, demonstrating the bias toward the focal hypothesis even in the absence of experimental manipulation. Additionally, both the focal and focal and explanation groups were willing to bet more on the focal team than the control group, demonstrating an increased motivation to gamble. By focusing on one event, alternative events fell to the wayside and consequently estimations of the likelihood of the focal event occurring and willingness to gamble on that event increased.

Hypothesis testing and confirmation bias affect not only betting but also affect judgments about the outcome distribution underlying a gamble. Consider the task used in Troutman and Shanteau (1977): two boxes of red, white, and blue beads in differing proportions, 70-30-50 (respectively) in one and 30-70-50 in the other, are shown. Then, not knowing from which box beads were drawn from, the participant observes sequences of beads drawn with replacement and then must estimate the probability that one of the particular boxes was being sampled from. Unbeknownst to the participants, the researchers controlled the content of the sequences and ordered the beads to be drawn in specific patterns of diagnosticity and nondiagnosticity. Participants' likelihood estimates demonstrated that irrelevant and nondiagnostic (e.g., blue beads, which were equally likely to come from either box) information were used to confirm prior hypotheses. However, in control conditions, the same participants demonstrated they were aware of the nondiagnosticity of the same information.

Overview of experiment

This experiment investigates how decision making agents with different experimentally-induced initial beliefs about outcome distributions update beliefs. Participants played two computer-simulated slot machines. For all participants, the machines displayed outcomes drawn randomly with replacement from the same hidden underlying outcome

distribution. However, only one group of participants viewed an accurate payout table, which displayed the machine's true outcome space. The other group of participants viewed a skewed payout table, which displayed the same payout values as well as additional (fictional) higher-value outcomes. Although all participants observed data from the same underlying distribution and fictional higher-value outcomes were not supported by any data, it was predicted that those participants who were shown a skewed payout table with fictional outcomes would maintain different beliefs about the underlying outcome distribution and outcome-generating process. Despite the lack of diagnostic evidence from the outcomes observed, the participants were hypothesised to persist in their belief in the fictional high outcomes.

The slot machines required a 5 pence stake for each play, which was taken from the £3.00 bank endowed by the experimenter to the participant at the start of the task. The payment of the stake was animated on the screen to emphasise the loss on each play. The machines used a random process to select outcomes from a fixed distribution: an outcome of 0, 2, 3, 4 or 5 pence with 17.4% probability or 10 pence with 13.0%. The expected value of a play at the machine was 3.9 pence, at a loss of about 1 penny given the cost to play. Participants played 80 mandatory pulls on the machines and answered questions before the start of play, after 30 arm pulls, and at the end after 80 arm pulls; machine choice and pace were up to participants but the number of pulls played was fixed.

Method

Design and materials

The experiment investigated the processes used by decision making agents for making inferences about hidden outcome distributions and outcome-generating processes. The design comprised a mixed design with one within-subjects factor over time and one between-subjects factor with two levels – participants shown an inaccurate, skewed description with fictional higher-value outcomes were compared to a control group who were shown an accurate description of the outcome space; no participants were shown the probabilities associated with the outcomes. Participants were randomly assigned to an experimental condition and remained unaware of any alternative task specifications. In each condition, participants were informed that their compensation would depend on the bank's value at the end of the task.

Payout tables were viewable before the task began and remained on the screen throughout the task except while the participants responded to questions. The tables showed the possible outcome values and the combinations of symbols required to obtain each outcome. For example, three lemons along the payout line won two pence. A line of the table also explained that if a play of the machine resulted in no matching symbols, the payout would be zero pence. It was emphasised in the instructions that these outcome values were exclusive of the five pence stake. Control participants were shown an accurate description viewed a payout table with the outcome values 0, 2, 3, 4, 5, and 10 pence and their associated symbol combinations. Participants in the group shown a skewed description viewed a payout table with the outcome values 0, 2, 3, 4, 5, 10, 15, 20, 25, and 100 pence and their associated symbol combinations. This skewed table suggested an initial belief that overlapped with the control participants' but also included false higher-value outcomes that would not be observed during the experiment. Despite the different payout tables, both groups played the same task and observed random draws from the same underlying distribution.

The experiment interface used fruit graphics, which are highly associated with slot machines imagery, and animations such as spinning reels and moving levers to simulate the appearance of real-world slot machines. The size of the screen display allowed the participant to see up to three symbols on each reel depending on the reel position. The machines had a single payout line (combinations of symbols must fall on the payout line to qualify for winnings) through the middle; symbols above and below ("near misses") were also viewable but were randomised. When a participant clicked on a machine to play it, the reels appeared to spin and then slow to a rest after three seconds, displaying the final screen of fruit symbol graphics that matched the outcome received on that trial. The final screen and numerical outcome value (e.g., 4 pence) was shown on the screen for 1.5 seconds. After this time, the screen returned to the initial state and participants could click on the machine they wished to play next. No bank or cumulative total information was shown to the participant at any time during the task except after completion.

At three times during the experimental task, participants were asked to illustrate their beliefs about what outcomes the machines paid out and how likely those outcomes were. To do this, participants were asked to articulate their beliefs using pie charts. This

method was chosen both for its familiarity for participants and for its convenience for eliciting comparable judgments between groups of the size of the outcome space. By providing blank response templates, this design requires participants to generate the outcome space and associated probabilities without being prompted by the experimenter for different outcome values. Alternative methods of elicitation of probabilities requiring prompting responses one outcome at a time or listing the full space of outcome values for the participant may have had undesirable consequences in defining the outcome space for the participant. Although phenomena such as subadditivity cannot be investigated in this paradigm, the benefits for the relevant hypotheses being tested are greater than this limitation.

To complete the pie charts, participants were given pen and paper with circle outlines, which served as blank templates. Each template sheet had one blank circle for the machine on the left and another identical one for the machine on the right, each with an indicator for the centre of circle to aid in drawing. Many participants readily understood this instruction but all watched the experimenter create an example pie chart and had the opportunity to ask questions.

Participants

Fifty-three participants were recruited to participate in paid studies on gambling from the University College London Psychology Department Subject Pool, Gumtree (a popular online UK notice board), and local newspapers. Data collection was run in two phases several months apart but no statistical differences between the collection groups were found (all p values > 0.21). One participant was removed from analysis because his responses indicated he did not use the pie chart templates correctly and two participants were removed from analyses of initial outcome distribution ratings due to the same problem; these participants recovered and were able to use the response templates correctly for the latter two judgments. Participants were randomly assigned to conditions, resulting in a final sample of 27 participants in the accurate payout table group and 26 participants in the skewed payout table group.

Procedure

In the first phase of data collection, this experiment was run on its own. In the second phase of data collection, this experiment was run in conjunction with three unrelated

experimental tasks on gambling. The tasks were administered in the same order for all participants; this experiment was run third in the series, following a task for which participants wagered on the roll of a die. It is assumed there are no carryover effects between experimental tasks. The procedure during the task is identical in both collection phases.

At the start of the task and at two times during the task, participants were asked to answer questions. Before beginning machine play but after being shown the playing environment including the machines and payout tables, participants were asked to give their best guess as to the hidden probabilities associated with the outcome distribution. After completing a pie chart for the machine on the left of the screen and another for the machine on the right, participants then began to play the machines. After 30 trials, the program automatically stopped the participants and prompted them to respond to questions. The experimenter presented the pen and clean paper and asked the participant to again illustrate the different payouts they believed the machines generally paid out, and how likely those payouts were. After 80 trials, the prompts were repeated. The timing of the prompts after 30 and 80 trials was not known to the participants. The payout tables that displayed the outcome space were visible during the first response time before play had begun but were not visible during the latter two judgment collections; participants completed these pie charts from memory.

After completing the 80 trials, participants responded to a forced choice question about the machines' outcome-generating processes. Participants were asked which statement most closely matched their belief: "playing required skill to avoid bad luck or bad streaks at machines" or "it didn't matter what I did or how I played".

Results

In this experiment, a mixed design compared two groups' changing beliefs over time about a hidden outcome distribution, measured by probability judgments and responses to direct questions after completion of the task. Neither group was compared to the true underlying distribution because it was believed that this standard is too high to be appropriate; an infinite number of underlying distributions might have produced the

sequences observed by the participants⁵. The experience of the two groups varied only on the range of outcomes listed in the payout tables shown during the task; all participants experienced slightly different sequences of outcomes due to the random outcome-generating process but no significant differences in the final sum of outcomes received were found between groups, $t(52) = 1.23$, ns.

Estimates of mean payout

Participants in both experimental groups made probability distribution judgments before play began, after 30 trials, and after 80 trials. At these collection times, participants were asked to report what payouts they thought the machine paid out in general, and how likely were those payouts. An example of a participant's response in pie chart form is shown in Figure 4.

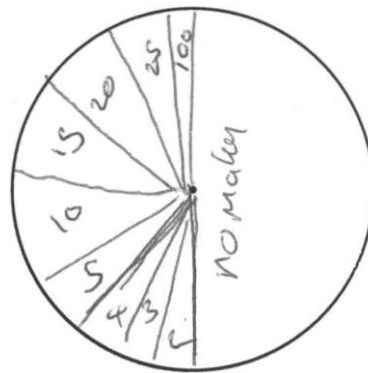


Figure 4. Example of a participant's hand-drawn initial judgment of a machine's underlying probability distribution of outcomes. This participant indicated that the possible outcome space included 0, 2, 3, 4, 5, 10, 15, 20, 25, and 100 pence outcomes and that the most likely outcome would be of no matching symbols, or 0 pence.

The first analysis of these data shown below in Figure 5 is of the raw mean estimates calculated using participants' pie charts. By measuring each pie chart segment, the researcher was able to assess how likely the participant believed each outcome to be and therefore the participant's subjective expected value for a play of the slot machine. In the example shown in Figure 4, the participant expresses a belief that the probability of an outcome of 0 pence is 50% and the subjective expected value of a play of the machine is 7.74 pence. Analysis of these data find that there are significant differences between the

⁵ For example, consider the cash register. If its premise and arithmetic were completely unknown, it would be impossible to learn arithmetic with certainty about its rules based on only observed outcomes (Oaksford & Chater, 2007).

two experimental groups ($F(1, 51) = 29.95, p < 0.001, \eta_p^2 = 0.38$) and within groups over the three judgment times ($F(1, 51) = 4.18, p = 0.02, \eta_p^2 = 0.08$). The interaction of the two factors is also significant ($F(1, 51) = 4.12, p = 0.02, \eta_p^2 = 0.08$). These findings suggest that participants who were shown skewed payout tables consistently gave higher estimates of the expected value of a machine play than control participants but showed a trend of converging toward estimates similar to control participants as a function of number of trials played. Similarly, control participants refined their estimates over time, as shown by the decreasing variance in the measure of mean payout over time. These data are illustrated in Figure 5.

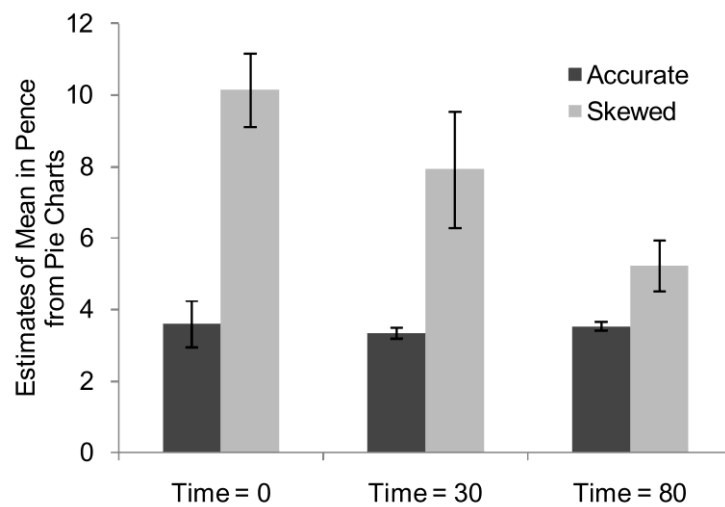


Figure 5. Data from participants' pie charts were used to calculate each participant's implied estimate of mean payout. Data shown in grey bars are from participants who were shown skewed payout tables including fictional higher-values of 15, 20, 25, and 100 pence. Responses for the left and right machines at each judgment collection time were averaged resulting in one response per participant for each of the three judgment times.

Remembering that each participant observed a different randomly-generated sequence of outcomes, the next analysis illustrated in Figure 6 examines each participant's pie estimates of the mean given their unique observed sequence of outcomes to assess whether group differences in estimates are due to differences in observed sequences. Their observed values were compared to their reported estimates from collection times after 30 and 80 trials to calculate an error measure; initial judgments are not included in this analysis because participants did not observe any outcomes before making initial judgments. These data confirm the analysis of the raw estimates: there are significant differences between the two experimental groups ($F(1, 51) = 7.06, p = 0.01, \eta_p^2 = 0.12$)

and within groups over the two judgment times ($F(1, 51) = 6.45, p = 0.01, \eta_p^2 = 0.11$). The interaction of the two factors is also significant ($F(1, 51) = 6.29, p = 0.02, \eta_p^2 = 0.11$).

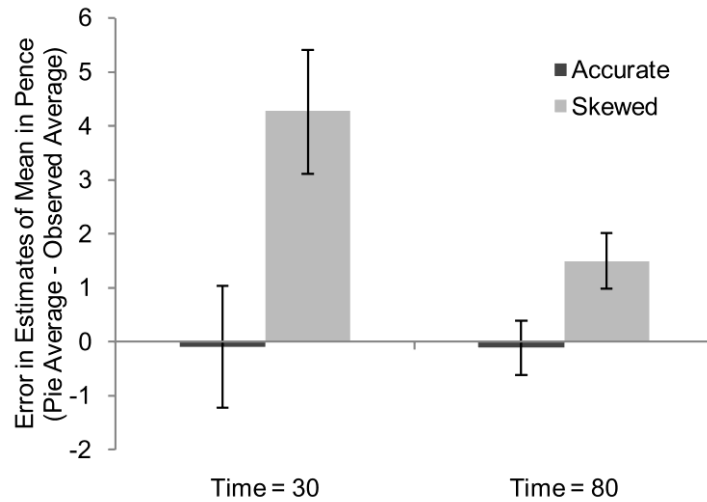


Figure 6. Each participant's mean observed payout for each machine was subtracted from each participant's implied estimate of mean payout for that machine to calculate errors in estimation. Values greater than zero indicate overestimation and values close to zero indicate accurate estimation. Errors for both the left and right machines were averaged resulting in one error measure per participant per judgment during machine play.

Accuracy of pie estimates

As shown in Figure 6, control participants who were shown an accurate payout table made precise estimates not significantly different from the observed data after 80 trials ($M = -0.09, SE = 0.13$; one-sample t-test against 0: $t(26) = 0.67, ns$) and after only 30 trials ($M = -0.10, SE = 0.13$; one-sample t-test against 0: $t(26) = 0.77, ns$). This accuracy provides support for the validity of this method of subjective probability elicitation to capture sensible data.

Representation of fictional higher-value outcomes

Although it is evident that the participants of the two groups perceive the expected value of each machine play differently, further analysis of the pie charts may explain where the difference emerges from. Estimates of the means alone cannot distinguish overestimation of the likelihood of observed outcomes (e.g., believing the 5 or 10 pence outcome happen more frequently than the observed data suggest) from categorically representing higher-value outcomes with any degree of likelihood (e.g., believing the 100 pence outcome is possible with a 1% likelihood). The pie charts show that the primary

source of the overestimation comes from maintaining a belief in the likelihood of the fictional higher-value outcomes. After 30 trials, 61.54% of participants in the skewed payout table group continued to maintain the unsupported belief of at least one higher-value outcome while only 3.70% (one participant) indicated the same in the control group ($p < 0.001$, Fisher's exact test). After even 80 trials, the difference in number of participants maintaining beliefs in higher-value outcomes remains significant (Skewed: 30.78%; Controls: 0%; $p < 0.01$, Fisher's exact test). This pattern shows that participants converged toward the observed data and no participants developed a skewed belief of unsupported fictional higher-value outcomes after having expressed a belief reflecting the observed outcome values only.

Beliefs about the nature of the outcome-generating process

Direct questions probed participants for their beliefs about the nature of the underlying outcome-generating process. It was hypothesised that participants who were shown the skewed payout table and expected to receive higher-value outcomes may believe that they are performing poorly on the task when they do not receive the expected outcomes. When asked whether outcomes required luck or skill or were merely random, only 3.70% of people (1 person) in the control group indicated that skill in avoiding bad luck was a significant factor while 30.78% thought as such from the group who viewed skewed payout tables. This analysis suggests that the different payout tables changed the participants' beliefs about the outcome-generating process. Indeed, the two are highly related: a logistic regression predicting belief type finds that those participants who categorically represent at least one fictional higher-value outcome are 8.00 times more likely to also believe the outcome-generating process is based on luck or skill ($B = 2.08$, $p = 0.02$). Although these correlations cannot provide directional explanations for participant responses, they support the hypothesis that beliefs about an underlying outcome distribution and associated underlying outcome-generating process may be related.

Discussion

It was hypothesised that expectations about outcome space would affect processing of evidence including the updating of beliefs about the outcome distribution and the nature of the outcome-generating process. In other words, despite seeing the same evidence, participants who were shown a skewed payout table including fictional higher-value

outcomes would persist in estimating the expected value of machine play to be higher than controls. This study's novel method of eliciting probabilities also enabled examination of participant's beliefs in probabilistic outcomes for understanding how these beliefs change over time.

Participants' overall ease of responding with pie charts to articulate probabilistic information indicates that decision makers are able to encode and use probabilities. Combined with the accuracy of the pie charts, decision makers seem to be capable of using probabilistic information successfully in judgments and decisions. Graphical models and representations may prove to be important tools for researchers.

As expected, participants' initial estimates of expected value of a machine play differed significantly: participants in both groups integrated the entire given outcome space into their beliefs, and those participants who were shown a payout table with fictional high values ultimately had higher estimates. Without any evidence to observe, participants used the payout table cues from the environment. Personal prior experience resulted in small variations between players in the relative likelihoods of different outcomes but the payout table information was consistently used by participants to define the scope of the outcome space.

After the presentation of more and more evidence, participants converged toward the observed frequencies regardless of initial beliefs. After only 30 trials, the proportion of participants representing these values decreased, and after 80 trials, the number of participants decreased still more. Although both groups converged toward the observed frequencies, the majority of participants who were shown skewed payout tables continued to represent these values in their subjective outcome distribution estimates. These data suggest that, despite a lack of any supporting evidence for the higher value outcomes, many participants continued to represent them in their subjective probability distributions. It may be the case that these participants eventually eliminate the higher-value outcomes but there may be individuals who maintain the beliefs despite even hundreds of data observations. A common model of the game that results in this pattern of beliefs is the gambler's fallacy whereby the absence of an outcome increases its likelihood of occurring in the future. As these data show, many participants who never observed a high value outcome still expected one in the future. However, the conclusions

to be drawn from these findings are limited by the nature of the random outcome-generating process; because fictional outcomes can never be disconfirmed by observed evidence in this paradigm, even a rational decision making agent might maintain such beliefs by, for example, weighting the prior belief in the likelihood of observing such outcomes relatively heavily compared to the weight attached to any observed outcomes. As discussed in Chapter 2 Study 2, the randomness element of chance game outcomes means that this problem of under-determination holds true for many popular games that are based on randomness.

Some might argue that these data support the case for including frequency information in payout tables and signage to increase punters' awareness and decrease their likelihood of updating beliefs without confirming evidence (Delfabbro, 2004; Gigerenzer & Hoffrage, 1995). But this solution confuses the issue: at the moment, there is no outcome probability information on payout tables of any format, frequentist or probabilistic. And it is possible that such information would serve to exacerbate the problem. Fully grasping the small chances of hitting a jackpot may make the problem of updating a continuous hypothesis of the likelihood of winning a jackpot from a random process even more difficult. Not winning a jackpot may become what is expected from playing the game and less diagnostic of a negative expected value game.

To return to the discussion begun in the previous study of this chapter, the present results provide corroborating evidence in favour of a model-based account of decision making for gambling. For the paradigm presented here, a decision making agent using heuristics might rely on representativeness or similarity to judge how high the payouts from a slot machine are. A rational decision making agent might calculate the expected value of the observed sequence and integrate these data with his prior beliefs. But neither type of reasoning and decision making predicts differences in agent understanding of the outcome-generating process. In contrast, understanding and updating beliefs about the outcome-generating process are the foundations of the model-based account. If an understanding of the outcome distribution is related to an understanding of the outcome-generating process, as the data suggest, then the heuristics and rational theories that do not account for outcome-generating processes may fall short of explaining decision making under uncertainty.

Study 5

A Computational Account of Model-Based Gambling

In the preceding studies, we have examined decision making in gambling in increasingly finer focus. We began by observing behaviour in the real world, then manipulating behaviour in the laboratory, and then eliciting online judgments from participants as they completed an experimental task. In the present study, I continue to refine this focus to investigate the mechanisms and processes that might drive these behaviours. It is my aim with this modelling exercise to demonstrate one possible route for developing a process-level account of model-based decision making in gambling.

Consider again the slot machine and the decision making agents who play them. What this problem and many real-world gambles and decision making tasks under uncertainty share in common is inductive inference, or prediction based on the observation of data. What will happen next? To make a bet on an event, gamblers typically integrate what is known and what has been observed to predict the outcome. Even with games such as roulette, in which the outcomes are independent, it is not uncommon to find gamblers studying previous outcomes to find patterns that will predict a winner.

But we are concerned with gambles for which the underlying outcome-generating processes are hidden. I assume that decision makers naturally and commonly approach these gambles as inductive inference tasks and use the tools of hypothesis evaluation to gather evidence to examine the hypothesis' fit to the data. There is a broad range of studies that support this view in other domains of everyday reasoning; for example: clinicians and medical diagnoses (Weber, 1993); mechanics and auto failure (Mehle, 1982); and scientists and research findings (Fischhoff, 1977). To close the chapter on decision making under fixed and unknown uncertainty, this study presents a modelling exercise that qualitatively explores the cognitive processes underlying hypothesis evaluation in gambling.

A model of hypothesis generation and evaluation, HyGene

As a process account of memory-based judgment and decision making, HyGene (R. P. Thomas, Dougherty, Sprenger, & Harbison, 2008) describes how hypotheses are

generated, evaluated, and used for probability judgments in inductive inference tasks within the confines of a bounded cognitive system. It is built upon MINERVA-DM (Dougherty, Gettys, & Ogden, 1999), which has been shown to account for many diverse judgment and decision making results including frequency judgments, conditional likelihood judgments, availability and representativeness heuristics, base-rate neglect, conjunction fallacy, the validity effect, and hindsight bias. It is a natural place to begin developing a computational account of decision making in gambling.

The original paper describes the model comprehensively and only the relevant details will be reproduced here. First I will give a brief overview of HyGene, describe the assumed structure of memory in which the model operates, and explain the process of retrieval from memory used in hypothesis generation and evaluation. Finally, I will discuss how this model relates to the model-based decision making account.

HyGene is a global matching, multiple-trace memory model. The model has three core tenets: data from the environment serve as cues for retrieval from long-term memory; cognitive and task characteristics limit the number of hypotheses that may be entertained at any point in time; and probability judgments are based on comparisons of only those hypotheses being considered at the time of judgment.

The structure of memory in HyGene

Memory comprises “hypotheses”, or mental representations of external events. Critically, one’s mental representation of an event is not assumed to be the same as the actual event. The collection of possible external events in the world (e.g., the universe of possible medical symptoms) is broken into smaller subsets, as illustrated in Figure 7. Within this structure, overlapping hypotheses are clustered near each other based on their degree of similarity. Only hypotheses that are known or have been experienced by the decision maker (e.g., symptoms seen or read about previously) can be represented and these are then further broken down to hypotheses that are relevant to the current problem (e.g., symptoms presented recently by similar patients or indicated by the current patient’s history). Of the hypotheses known to the decision making agent, only a few may enter working memory, the set of leading contenders (SOC), for consideration at any given time.

The contents of episodic and semantic memory are traces consisting of data, hypotheses, and context. These traces are represented as vectors in the computational model, enabling a straightforward comparison of similarity between vectors on features. For example, context, which refers to any information that might be encoded as peripheral to the hypothesis and data, is a feature within a vector and is represented by a value that is numerically similar (in varying degrees) to other hypotheses; hypotheses that share the same context also share the same feature value.

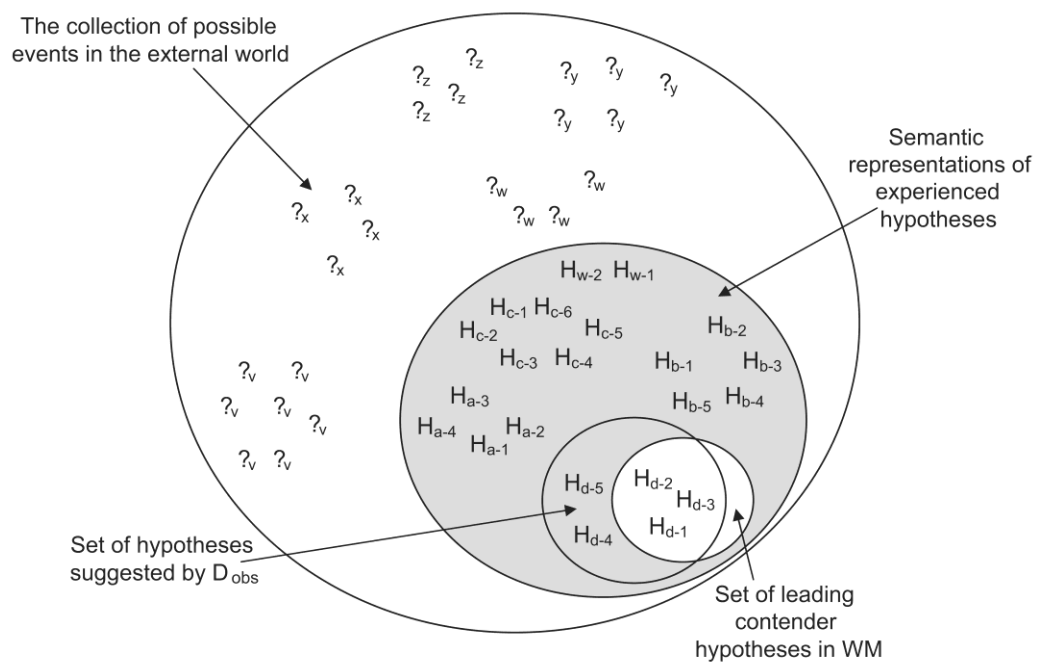


Figure 7. A Venn diagram of the semantic structure assumed in HyGene. The largest circle represents the entire space of possible outcomes including those ideas unknown to the decision maker (elements with ?s). The large grey area represents the decision maker's knowledge (elements with Hs). The larger circle within the grey area represents the decision maker's relevant knowledge, as cued by observed data, D_{obs} . The smaller white area of working memory represents the hypotheses currently being considered (R. P. Thomas et al., 2008, p. 156).

Hypothesis generation and evaluation

To access long-term memory, a probe vector is created. Traces in memory that are activated based on their similarity to this probe are then extracted to the SOC as leading contender hypotheses, replacing any weaker hypotheses that might already be under consideration. This process of probe and extraction to working memory – the generation of hypotheses – continues until the decision making agent fails to generate new hypotheses on successive attempts. If the evidence in favour of a hypothesis is extremely

strong, the decision maker may generate only the one hypothesis, as no others will surpass the necessary threshold. Then, the posterior probabilities of hypotheses are evaluated by comparing the hypotheses' base rates from episodic memory. Hypothesis-guided search follows, whereby leading contender hypotheses guide cue selection.

As a starting point for modelling decision making from a model-based approach, HyGene shares with model-based decision making many of the same principles.

Mental representations—Both accounts of decision making distinguish between external events and the decision making agent's internal representations of those events. Through subjective interpretation and forgetting, internal representations differ from the observed data.

Limitations of the cognitive system—Both accounts acknowledge that the space of possible states known to the decision making agent and the number of hypotheses that can be simultaneously entertained are limited compared to a rational decision maker due to cognitive and task characteristics.

Cascade of errors throughout the process—Both accounts predict that errors in any one phase of decision making will have knock-on effects on other phases in the process. For example, an error in hypothesis generation is reflected in probability judgments.

There are, however, several critical disagreements in theory between the model-based approach and HyGene. These differences can be illustrated using a coin.

Consider a sequence of coin flips. After three coin flips, the sequence runs HTH. After seven coin flips, the sequence might run HTHTTTH. And so on, for any length of sequence imaginable. Two questions commonly asked about coin flips in a gambling context are: what is the likelihood that the coin is fair? And what will the next coin flip outcome be?

How should a computational model of decision making in gambling approach this problem? We know that coin flips are random and may result in different sequences, with fair coins doing so with equal probabilities and biased coins with lower probabilities.

Representing each “external event” (or sequence) in memory would likely absorb a significant amount of resources with similarity clustering based on arbitrary criteria. Would similarity be based on the number of heads in the sequence? The alternation rate? But the data from, and consequently the similarities among, random processes are rarely diagnostic. When faced with randomness, to study the outcomes is to focus on the distractions. The underlying process generating the outcomes is the real prize. Even using base rate information from episodic memory may lead the decision making agent astray, making biased inferences from small samples. Instead of mental representations of external events as described in R. P. Thomas, et al. (2008), model-based decision making assumes mental representations include higher-order structural information about the underlying outcome-generating process. Rather than represent the sequences of 3, 300, and 3,000 coin flip outcomes and the similarities among them, a decision maker represents coins with probabilities of landing on heads of 0.5, 0.6, 0.7 and so on.

This coin flip example illustrates two key theoretical differences.

Content of mental representations—In HyGene, mental representations are understood to be mental representations of *events* that have or have not yet been experienced. As described in R. P. Thomas, et al. (2008), examples of these representations might be symptoms or test results presented to a doctor for diagnosis. Although these representations may contain an arbitrary level of detail in the computational model including context features, this picture of a mental representation is incomplete. As discussed in the introduction of this chapter and in the results of the previous studies presented, there is strong evidence indicating that decision makers’ beliefs about events contain higher-order structural information such as the causal processes that led to the outcome and this structural information is reflected in how events and hypotheses relate to each other.

Similarity mechanism of evaluation—A mechanism of strength of association or similarity between and among data and hypotheses falls short of capturing the intuitive and rational representations of randomness, which is essential to the study of gambling. This follows as a direct consequence of the higher-order structure underlying the internal representations of events (as discussed in Chapter 1: Schemas).

An experimental paradigm similar to that used in Study 4 of this chapter may provide an appropriate test of whether the model-based assumptions are necessary or HyGene's similarity comparisons are sufficient. In Study 4, analysis found that the two experimental groups—those who were given an accurate prior hypothesis about the game and those who were given a similar but inaccurate prior hypothesis—maintained different beliefs about the underlying outcome distribution and outcome generating process, as the group with inaccurate prior knowledge was less likely to respond correctly to judgments collected during the experiment. Over time, however, these participants converged toward the observed data. Critically, this shows that a decision making agent may at any time conclude that the observed outcomes and probability distribution are the true underlying outcomes and probability distribution, without any inferences going beyond the data.

Whether these same differences in theory and behavioural data are borne out in the modelling exercise here may provide some support for whether future computational work on model-based decision making will need to account for these differences.

Overview of the study

This study aims to provide a qualitative exploration of decision making when prior knowledge does not match observed data. For ease of interpretation throughout this study, I will simplify this manipulation: a group with correct prior knowledge compared to a group with incorrect prior knowledge; however, in the model's terms, this manipulation translates to comparing a group with an accurate and narrowly-defined set of hypotheses to a group with a broad diverse set of hypotheses. These findings should also apply to the more general problem of mismatch between observed data and expectations. The contents of semantic memory and the retrieval probe vector as well as experience were manipulated. The data produced are compared to simulations run according to the specifications in Simulation 1 of R. P. Thomas, et al. (2008).

Simulation methodology

To create a manipulation in the model that is qualitatively similar to that used in the experimental paradigm of Study 4, the content of memory and the probe vectors used for retrieval were modified. Semantic and episodic memory structures are two components of memory which are conventionally modelled as associated, with semantic

structure building upon episodic traces. In this simulation, the creation of the two was split into separate processes and completed in parallel. Both groups of simulation participants observe the same data (same episodic content) but have different prior distributions of beliefs (different semantic content).

To achieve this, the vectors used as the basis for memory were modified. In HyGene, a random vector is created and copied (with degree of similarity as a parameter) to create prototypes that serve as the basis of episodic and semantic memory. One of these prototype vectors is designated as the focal hypothesis while another is designated as an alternative hypothesis; which hypothesis is the “focal” one is simply for the purposes of evaluation and probability judgment at a later stage. The traces from these prototypes make up episodic memory. Importantly, the modifications in this study did not change the content of episodic memory; both groups observed the same data.

For the participants who enter the task with the correct prior knowledge, only variations on one hypothesis were considered, rather than many alternative hypotheses. In other words, these participants had a narrowly-defined idea about how the game worked and entertained only adjustments to that fundamental idea. This does not necessarily assume that the strength of the focal hypothesis is particularly strong, only that the set of hypotheses considered are all highly similar to each other. This manipulation of a narrow structure of semantic memory mimics a confirmation bias in hypothesis testing, whereby alternative hypotheses are not considered and incoming evidence is interpreted as supporting the focal; this has been shown to be the prevalent strategy in many learning and prediction tasks (Fischhoff & Beyth-Marom, 1983; Klayman & Ha, 1989; Nickerson, 1998). In the model, the content of semantic memory is reduced to only variations on the focal hypothesis; see the right panel of Figure 8 for an illustration. This manipulation is represented by participants having only one type of hypothesis in mind. With all memory traces emerging from the same prototype, the scope of hypotheses in semantic structure is more focused.

For those participants who enter the task with incorrect prior knowledge, a broader range of alternative hypotheses is considered, which includes, but is not limited to, the true focal hypothesis. This is represented in the left panel in Figure 8. This specification

is the original, broader, representation of hypothesis generation depicted in R. P. Thomas, et al. (2008) and approximates the manipulation of prior beliefs in Study 4.

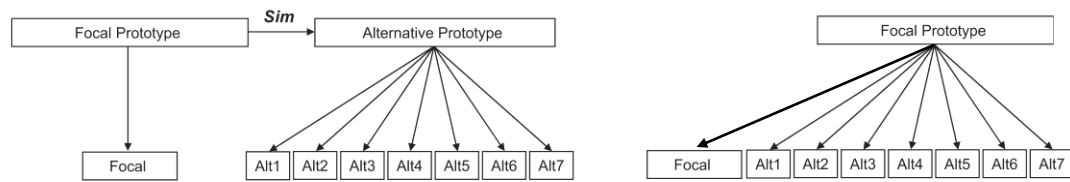


Figure 8. The left panel is a reproduction from R. P. Thomas, et al. (2008) showing how semantic memory is structured in HyGene. Semantic memory is based on both a focal hypothesis and alternative hypotheses. This specification is used in the present modelling exercise to approximate a group of participants who enter the task with a broad range of hypotheses. The right panel shows a modified version of semantic structure in which there are only variations on the focal hypothesis. This specification approximates those participants who enter the task with narrow prior knowledge.

Results and discussion

Two hypotheses are tested for qualitative comparison to experimental data presented in Study 4 of this chapter. The prior knowledge of one group was designed to approximate a mismatch with observed data (e.g., misunderstanding the task or having no knowledge of the task, using the original specifications from R. P. Thomas, et al. (2008) Simulation 1) and a second group was designed to match prior knowledge and observed data (e.g., having a fine focus on what will be observed in the task). Also, for both groups, experience was manipulated to approximate the progress of learning over time. In interpreting the results shown, it is helpful to first note that the parameter *Sim*, which is the measure of similarity between the focal hypothesis and alternatives in semantic memory, was set relatively low at 0.5; in models that include alternative hypotheses, this parameter ensures those hypotheses appear dissimilar to the focal hypothesis. As *Sim* approaches 1.0, the similarity between the focal and alternative prototypes would decrease and any differences between the experimental groups would diminish.

Simulation results are shown in Figure 9. The figure plots the probability of choosing the correct hypothesis as a function of encoding fidelity across different distributions of base rates for the hypotheses in semantic memory. Within each graph are series for data simulated with increasing levels of experience. These graphs are divided into two panels, which show the results for a focused decision maker with correct prior knowledge (top

panel) and the results for a decision maker with incorrect prior knowledge (bottom panel).

Match between semantic memory and observed data

Comparing the profiles of the data series in the top and bottom panels of Figure 9 shows a clear difference in the likelihood of choosing the correct hypothesis due to the content of semantic memory. In the bottom panel, participants with incorrect prior knowledge are highly likely to choose the correct hypothesis. In the top panel, the result varies depending on encoding fidelity. Because the participants with incorrect prior knowledge have a broader range of dissimilar hypotheses in semantic memory, they may be more easily able to identify the true focal hypothesis. Because participants with correct prior knowledge have a narrow range of alternative hypotheses in semantic memory that are highly similar to the true focal hypothesis, it may be difficult to identify the correct hypothesis.

Recall that in Study 4 of this chapter, participants who entered the task with the correct prior knowledge did well on judgment tasks in which they were asked to estimate the long-run average payments and outcome distributions of the slot machines played, while participants who entered the task with incorrect prior knowledge significantly overestimated average payments and included false outcomes in estimations of the outcome distributions. Qualitatively, these simulation results do not match the empirical data collected in experimentation with actual participants.

Experience

Within each graph are plotted the data from simulations run with varying levels of experience. The low experience decision maker has fewer than 20 traces stored in episodic memory while the high experience decision maker has approximately 500 traces stored. In every condition, there are no gains from experience. In the top right graph of Figure 9, the high experience decision maker is less likely to choose the correct hypothesis compared to the low experience decision maker. This counterintuitive result likely occurs because, when experience is low, the strong focal hypothesis is relatively less affected by the encoding loss or forgetting while the weak alternative hypotheses suffer greatly with such few episodic traces.

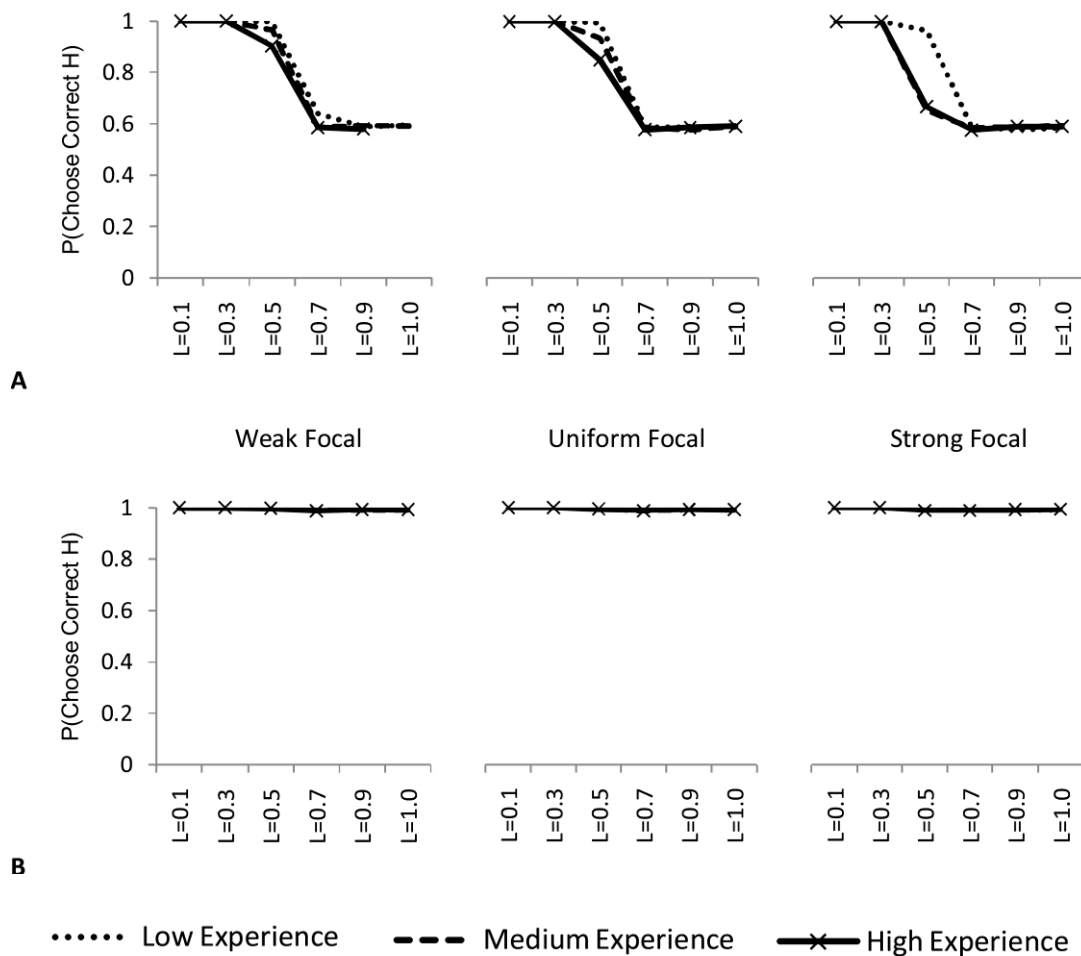


Figure 9. Probability of choosing the correct hypothesis as a function of content of prior knowledge (top panel A = narrowly-defined semantic structure with high degree of match to observed data; bottom panel B = broader semantic structure with mismatch to observed data), encoding fidelity (L), expertise (1x, 10x, 30x episodic traces), and relative strength of the focal hypothesis (weak focal = 1-6-2-2; uniform focal = 2-2-2-2; strong focal = 10-2-2-2). In all simulations, Sim = 0.5 and number of participants = 2000. Bars for standard errors of the means are not shown but are all below 0.0067.

Again, this is not the result expected based on the data collected in experiments. While participants in the experiments converged over time to the observed data, the participants in the simulations are unchanging. In HyGene, “experience” is approximated by increasing the number of episodic traces in memory. Given the similarity mechanism for choosing among hypotheses, increasing the episodic traces in memory serves to amplify any effects found. In the case of mismatched priors and data, there may be a ceiling effect. In the case of matched priors and data, increased experience serves to push the already-similar hypotheses closer together; the true focal hypothesis is harder to identify.

Encoding fidelity

Figures shown in the bottom panel B replicate the results depicted in Figure 5 of R. P. Thomas, et al. (2008, p. 168), with the probability of choosing the correct hypothesis high and unvarying as a function of encoding fidelity. The probability increases slightly due to decreased encoding fidelity, or randomised information loss, across all hypotheses, which creates differences between the otherwise similar hypotheses (M. R. Dougherty; personal communication, October 11, 2010). In other words, the model finds that decision makers who enter the task with many alternatives in prior knowledge are highly likely to choose the correct hypothesis independent of ability to correctly encode the data.

Figures in the top panel A show the data from the modified model in which a narrow semantic memory closely matches observed data. Again, the main effect of encoding fidelity is present, whereby a low encoding fidelity results in high likelihood of choosing the correct hypothesis. However, the simulations show that as encoding fidelity approaches 1.0, the hypotheses in semantic memory converge (the hypotheses are all variations of the same vector) and the model is less likely to choose the correct hypothesis.

The simulation results indicate that HyGene in its present form is not a computational account of gambling and model-based decision making and cannot fit the empirical data from Study 4. Despite addressing numerous findings in the judgment and decision making literature, its theoretical basis in similarity of events restricts its ability to represent perceptions of and judgments about randomness. Because similarity is an arbitrary measure for assessing randomly-produced outcomes, the basic principles of HyGene and other similarity-based models of decision making, are flawed from the outset for describing gambling behaviour. This simulation also shows that HyGene does not consider learning about underlying outcome-generating processes. The studies previously presented in this thesis have shown that individuals use structured representations of the outcomes and the outcome-generating processes of the problems they face. The model's inability to qualitatively predict improvements in judgment due to learning highlights the absence of a capacity to evaluate the evidence being observed.

There may be cause to expand HyGene's notion of a hypothesis to include higher-order structural information, whereby clustering in memory is based on not only similarity but also meaningful structure. This modification may address both concerns identified in this study. Future progress in a psychology-driven computational account of model-based decision making may benefit from the advances already made in machine learning. Although the premises and goals of the two approaches are different, there are shared obstacles. Researchers of adaptive control also study algorithms for sequences of decisions under fixed and unknown uncertainty (Burghes & Graham, 1980; Kaelbling et al., 1996; Sutton & Barto, 1998). For example, Sutton's Dyna architecture (Sutton, 1991) builds a model based on data and then adjusts behaviour using both data and the model.

Chapter 4

Dynamic Uncertainty

Unfortunately, a coin for illustrating dynamic uncertainty does not exist. If it did, it would spontaneously gain and lose additional sides of varying heaviness. Those observing the coin flip might not know what outcomes, let alone likelihoods of those outcomes, existed, what process would generate those outcomes, or what parameters might predict them. In other words, the coin would behave much like people do.

Environments of dynamic uncertainty present problems for agents that may be incompletely described or entirely misunderstood. A decision making agent faced with such a class of uncertainty has incomplete information about both the underlying outcome distribution and the outcome-generating process. Further to this ignorance, the distribution and process may change over time and the reasons for such state changes are also unknown to the agent. Unlike in the games previously discussed such as dice, an agent in dynamic uncertainty does not know what parameters, outcomes, or actions are relevant. To make even slight progress, the agent must learn about the processes underlying the outcomes—using his prior beliefs, the description of the problem, and the incoming evidence.

Building on the discussion of beliefs about future events from Chapter 3 in which beliefs about the likelihood of future events were shown to play a role in judgment under uncertainty, the present chapter continues under this direction and explores how decision making agents use and update beliefs given evidence. In gambling, as in many contexts of everyday reasoning, events unfold over time with evidence emerging sequentially and not always chronologically. For example, when playing poker, bets are made and cards are dealt in sequence around the table and an opponent's bet might reveal information about cards dealt earlier in the hand. Furthermore, as in the poker example described, the order that evidence is revealed may have no meaningful relationship to the underlying outcome-generating processes. The sequential nature is especially critical under dynamic uncertainty when new evidence may indicate not only adjustments to hypotheses but also qualitatively different hypotheses than previously considered.

Updating beliefs under uncertainty

A necessary part of repeated and sequential decision making under uncertainty is taking in new evidence and updating beliefs. The process of belief updating begins with prior beliefs that are then adjusted conditionally on the observation of new evidence. In other words, prior beliefs and expectations are the starting point from which subsequent judgments are generated. They may affect how a decision maker goes about updating his beliefs.

In rational accounts of belief revision, a decision making agent systematically and fairly evaluates new evidence and integrates it with prior beliefs in such a way that considers the full set of evidence under consideration. As discussed in Chapter 1, Bayes' theorem is a general mathematical rule that results in a posterior probability expressing the degree of belief about the probability of a hypothesis being true after observing data. The fundamental principle underlying this logic is the competition between hypotheses, and Bayes' theorem favours the hypothesis that is most likely relative to others.

In heuristics accounts of belief revision, the process is described as anchoring and adjustment: focusing on a value, which may be irrelevant to the task, and then incrementally, intuitively adjusting from that reference point to reach an estimate (Tversky & Kahneman, 1974). The heuristic has been identified in gambling scenarios to the same result as in other domains: adjustments from the initial anchor are insufficient (Chapman & E. J. Johnson, 1994; Schkade & E. J. Johnson, 1989). Several processes have been proposed to drive or explain this empirical finding. The insufficient adjustment may be due to satisficing after reaching a plausible estimate (Epley, Keysar, Gilovich, & Van Boven, 2004). Alternatively, decision making agents may seek and construct reasons to explain (and perhaps justify) beliefs, rather than using preferences and beliefs to inform reasons (Shafir, Simonson, & Tversky, 1993). Indeed, Nisbett and Wilson (1977) show that people may be unaware of the reasons for their choices and decisions at the time of judgment, and subsequently generate explanations when asked. In addition, Epley and Gilovich (2001) have shown that the source of the anchoring value affects the degree of adjustment. Errors in the choice or estimation of an anchor for a judgment may cascade to subsequent judgments even if the initial anchor is later discounted. Such a tendency for constructing a cohesive narrative post-hoc suggests that

initial biases and errors in prior beliefs may persist in online and global judgments despite disconfirming evidence.

In this chapter, I integrate the ideas and results of the previous two chapters to explore how the different ways in which agents perceive and reason about problems of uncertainty affect decision making and learning. I present data from two new studies. In the first study, prior beliefs were experimentally manipulated to test how subjective perceptions of the problem (also known as prior beliefs) of different informative values are integrated with incoming evidence. In the second study, the diagnosticities of focal and peripheral learning cues were manipulated to study how decision making agents learn about hidden outcome-generating processes.

Study 6

Revisiting the Endowment Effect from the Starting Gate

Although humans are the archetypal examples of problems of dynamic uncertainty, the complexity of human behaviour (and the motives and emotions that implicitly accompany it) can make research into decision making about people challenging. Horses, on the other hand, can be similarly unpredictable but are qualitatively simpler. Horse races have been used previously in studies of decision making in gambling, by Metzger (1985) which demonstrated the applications of decision making findings in racetrack betting markets, finding that individuals' behaviour aligned with the predictions of the gambler's fallacy, risk preferences changing as a function of reference points, and the illusion of validity that had been found previously in the laboratory.

Before the race begins, gamblers must integrate their prior beliefs to converge on an initial wager without any evidence from the present race. There exists a large volume of data—about the horses, the jockeys, the pitch, the weather and more—of high and low predictive validity available before a given race, but there is no clear formula for integrating these data into a meaningful judgment⁶. How does a punter wade through these data, choosing which to weigh heavily and which to discard? Ceci and Liker (1986) found that even non-expert racetrack gamblers used complex multivariate models to predict winners and post-time odds. Furthermore, these initial beliefs may have durable effects on the processing of subsequent incoming evidence. Believing the track and running conditions to be poor may influence judgments of a losing horse's true ability. Prior beliefs are critical in the development of judgments over time.

The endowment effect

One of the most robust and widely-reported anomalies in economics is a pattern of preferences called the endowment effect, whereby people require more to give up an object than they will pay to acquire it (Thaler, 1980). In other words, being given an object to own or merely touch changes the way in which people subsequently value it.

⁶ This has been done successfully by modelling large volumes of historical data but, unlike the guaranteed-win systems of roulette, this model is not for sale (personal communication, J. E. V. Johnson, November 6, 2008). Interestingly, there is also a real possibility that no amount of data will predict a result; e.g., if a horse race is fixed, as in India (personal communication, S. Puri, May 7, 2010).

Being endowed with an object is not unlike the common experience of being endowed with a belief – such belief endowments occur in the real world implicitly through advertising and explicitly through advice, for example. The endowment effect has been identified in chimpanzees (Brosnan et al., 2007) and in humans for markets ranging from chocolate bars (Kahneman, Knetsch, & Thaler, 1990) to lottery tickets (Bar-Hillel & Neter, 1996). People are reluctant to exchange a lottery ticket for more than its expected value if it is endowed. Will gamblers inflate the value of endowed beliefs in the same way?

Other demonstrations of such trades have found that we do not always find value in endowments when there is none. Brookshire and Coursey (1987), in searching for evidence of learning, find that over repeated market experiences the endowment effect diminishes and participants' valuations for willingness-to-accept and willingness-to-pay for environmental goods begin to converge. Gamblers may be able to learn from evidence to overcome initial endowed beliefs. However, some gambles cannot be repeated. Rational individuals must learn both across markets and within a single market.

The impact of evidence source credibility on prior beliefs

The source of a belief or item of evidence should be considered when evaluating it. At the racetrack, a tip from a business partner would take precedence over an unsolicited tip from a stranger. However, there are several examples of decision makers using source information to their disadvantage. Schum (1981; Schum & Martin, 1982) found that, when asked to integrate vast quantities of disparate data, people may inadvertently use disconfirming and nondiagnostic evidence as corroborating evidence, due to considerations of source. For example, advice from an expert, which might appear relevant, may be accepted despite its true irrelevance to the wager at hand (Kahneman & Tversky, 1973; Metzger, 1985). Harman (1986) suggests that revisions to prior beliefs may rely on several principles, including coherence with other prior beliefs and, similarly, an interest in not being inconsistent. When experts and typically appropriate sources offer information, decision makers happily integrate the information while maintaining consistent and coherent explanations for their beliefs.

Although such information from experts or less credible sources may influence initial wagers, it would be expected that subsequent wagers for which the decision maker

himself can evaluate incoming evidence would dominate any so-called advice. Consistency and accuracy of beliefs are critical for gamblers, but profit is the ultimate criterion.

Overview of experiment

This experiment uses horse race wagering to explore endowed beliefs and the impact of belief source on belief updating over time. Participants' prior beliefs were experimentally manipulated in the following ways. First, rather than be given tangible and salient endowments, participants were endowed with beliefs but no ownership and, in some cases, no physical evidence of endowment. Second, the type of belief endowed was manipulated between subjects: one group received a valuable tip and another a meaningless instruction as to which horse the participant "likes to win". And third, participants were probed multiple times for online judgments within single markets. Unlike previous experiments that collected only one-off judgments or a series of one-off judgments from repeated markets, this dataset includes beliefs updated upon evidence and feedback. As in traditional endowment experiments, participants were offered the same wagering choices with no costs for switching or trading of items. No horses were owned and responses as wagers clearly represented choices of money and reward rather than the endowed items.

It was predicted that the manipulations of source of belief and judgments over time would divide the rational normative and heuristic theories from the model-based account of decision making. First, as has been found in previous studies, the value of the source of belief was predicted to have no effect on wagering, in contrast to the normative theory. Whether participants were shown a valuable tip or a valueless prompt to like a horse, they would show a bias in favour of the horse. Second, the biases in initial beliefs were predicted to endure to subsequent judgments collected online during the task, despite observing evidence disconfirming endowed beliefs, in contrast to the heuristic account. Such a result again would indicate that a structured approach to understanding decision making is necessary.

Method

Design and materials

The study comprised two races. The first race used a between-groups manipulation of source of prior beliefs: one group of participants was instructed that they “liked a horse to win” but were not provided with any evidence or reason to favour that horse, a second group was given advice from a local track “tipster” that a horse would win; both groups were compared to a control group of participants who received no special instructions. Participants were randomly assigned to an experimental condition and remained unaware of any alternative task specifications. A second race served to replicate the first with a separate independent horse race under the same instructions and assess the transfer of learning from feedback.

All participants completed the same task, including wagering on the same horses (the first and second place finishers of each race; place information was withheld from the participants during the race), and received the same instructions except regarding the specific manipulation of endowment. Participants in the control group read: “You will be assigned to two of these top [the field was narrowed to those horses finishing in the top four] horses. While you’re watching the race, we’ll ask you to stake money on these two horses.” Participants in the “like”-endowment group read: “You will be assigned one of these horses. Then, we’ll match your horse against another of these top horses. While you’re watching the race, we’ll ask you to stake money on these two horses” and saw a script reading “You like Happy Daze [2nd place horse] to win” before the race began and a “Your Horse” identifier at wagering prompts. No reasons were given for why the participant should favour the horse. In contrast, participants in the “tip”-endowment group were shown the same instructions as those in the control group, and shown a pop-up window containing the advice: “The tipster at the track has said Happy Daze [2nd place horse] looked eager just before going into the gate.” Participants in both experimental groups were endowed with the same second-place finishing horse and observed the same early disconfirming evidence. Participants were free to stake any wager amount on either horse at all wagering prompts. Participants’ payments were based on the outcomes of a random selection of these bets, incentivising participants on each trial to back the horse they believed would ultimately win.

Race footage was taken from the Lagoon Games complete horse racing night game package, which included video of ten genuine horse races. For this experiment, the first race (the Lagoon Handicap Hurdle; two miles and one furlong) and third race (Lagoon Classic; 7 furlongs) were shown. The races comprised new horses about which the participants had no performance history or odds information. Audio commentary was muted.

Participants

Forty participants were recruited to participate in a paid study on gambling from the University College London Psychology Department Subject Pool, Gumtree (a popular online UK notice board), and local newspapers. One participant's data are omitted from analysis due to computer error at time of data collection. Participants were randomly assigned to conditions, resulting in a final sample of 13 participants in each of the control group, "like"-endowed group, and "tip"-endowed group.

Inventories were administered after the task to measure numeracy and gambling problem severity (see Appendices 1 and 2). This sample of participants scored 85.80% (SE = 3.31%) on the numeracy scale, where 100.00% reflects a perfect score and scored 3.46 (SE = 0.67) on the gambling problem severity scale, where scores above 2.00 indicate possible problems with gambling.

Procedure

This experiment was run in conjunction with three unrelated experimental tasks on gambling. The tasks were administered in the same order for all participants; this experiment was run first in the series. It is assumed there are no effects of the other experimental tasks.

At the start of the task, participants were seated at a computer with instructions for the task according to the randomly assigned condition. Participants were instructed they would be viewing horse races that had happened in the past and wagering on which horse would win. Wagers would be prompted before the race began and at several points during the race when the video would be paused; each wager would be new and independent of other wagers. Participants were further instructed that each wager was

equally important because payment would be calculated using a random selection of all wagers.

The endowment manipulation occurred before the first wager of the race. Participants in the “like” group viewed a screen with the horse names and jockey colours and read that they liked a particular horse to win. Participants in the control group viewed the same screen without the additional script. Participants in the “tip” group viewed the same screen as control group participants and also a pop-up window with the tipster’s advice. The prompt for wagers showed the two relevant horses’ names and sliding response bars ranging from £0 to £10 virtual money; participants were instructed to place £5 on each horse if they thought each was as likely to win as the other or more or all of the £10 on the horse they thought was more likely to win. The responses were linked such that if the participant indicated a wager on one horse (e.g., £8 in favour of the horse), the wager for the other horse would immediately register as the remainder of the £10 (£2 in favour of the horse). When the participant submitted his wager, the response bars were cleared and wager response history was not shown at subsequent wager prompts. During each wager prompt, the paused video remained on screen and the current positions of the horses were listed.

Wagers made before the start of the race probed the participant for his a priori preference for which horse would win, without data from the horses’ performance during the race. Wagers were collected at five additional points during the race, resulting in a dataset that reflected each participant’s repeatedly updated beliefs about the horses.

Results

To test the effect of endowment on wagering, a between-subjects design compared the wagering biases of three groups which differed only on the source of their prior beliefs. The repeated wagers design enables an examination of the duration of the endowment effect over time. The second race serves as a replication and tests the durability of any effects for carrying over to a second race after feedback on the first race. Although both Race 1 and Race 2 use similar experimental designs, the dynamics of the horse race stimuli differ between races; this is reflected in the differences in the overall betting profiles between races. Consequently, analysis of participant wagers is shown below in

two separate sections. Data from the “tip” group is excluded from analysis in Race 2 due to the tips being spoiled, from the participants’ perspective, after the loss in Race 1.

In initial piloting experiments, a small yet significant screen position bias was found; the horse for whom the response bar was positioned on the left of the participant’s screen garnered higher wagers. Screen position was randomised and recorded in this experiment, and controlled for in each of the following analyses.

Race 1

Initial wagers—In a regression analysis predicting initial wagers, “like” participants wagered more than control participants ($M = \pounds 6.42$, $\beta = 0.30$, $p = .08$; controls: $M = \pounds 5.23$) and “tip” participants wagered significantly more than control participants ($M = \pounds 6.57$, $\beta = 0.38$, $p = 0.02$). In the same model, numeracy was also a significant predictor ($\beta = -0.37$, $p = 0.02$); in other words, participants with higher numeracy scores wagered less.

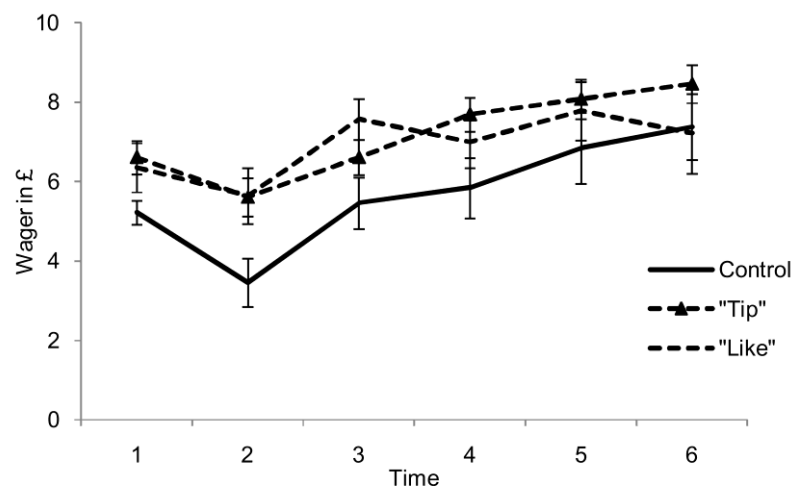


Figure 10. Mean participant wagers (and standard error bars) for the second place finishing horse, also the endowed horse in the two endowment conditions “Tipped” and “Like”, are shown. Data at the first time point represent beliefs before the start of the race.

Wagers over time—A mixed ANOVA, considering experimental condition, screen position, and numeracy score, confirms that there are differences between groups over time $F(2,32) = 4.57$, $p = 0.02$, but no significant interaction between time and experimental condition; in other words, an endowment before the race does not alter how beliefs are updated over time ($F(7.63,115) = 0.91$, ns). These data are shown in Figure 10.

To test whether the endowment effect is merely an initial heuristic overcome with the presentation of data or an enduring bias despite the presentation of data, the mean wager difference between groups was tested for time periods after the race had begun. The difference between groups remains significant even after the race began ($F(4, 34) = 4.99$, $p < 0.01$). Both endowment group participants' wagers were inflated compared to control participants' wagers at the second time period ("like": Beta = 0.52, $p < 0.01$; "tip": Beta = 0.44, $p < 0.01$) suggesting that the endowment effect endured despite observing the same evidence as controls regardless of the source of the prior belief. However, only "like"-endowed participants' wagers continued to be larger than controls' in the following time period (third time period: beta = 0.54, $p < 0.01$).

Tie-breaks—Wagers were prompted at one point in time during the race when the two horses were tied in the same position. This tie point occurs at the second wager after the race had begun (third time period in Figure 10). At this point in the race, the "like" group still wagers significantly more money in favour of the endowed horse compared to controls (Beta = 0.54, $p < 0.01$) while the "tip" group wagers no differently from control participants.

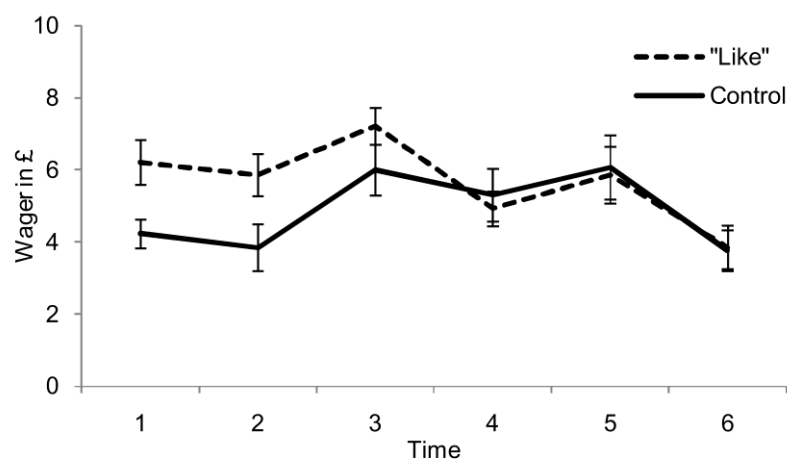


Figure 11. Mean participants' wagers (and standard error bars) on the second place finishing horse in Race 2 are shown.

Race 2

Initial wagers—In a regression analysis predicting initial wagers, "like" participants wagered significantly more than control participants ($M = \pounds 5.92$, beta = 0.44, $p = .03$; controls: $M = \pounds 4.23$). In the same model, numeracy was not a significant predictor.

Wagers over time—As in Race 1, a mixed ANOVA considering experimental condition, screen position, and numeracy score confirms that there are differences between groups over time ($F(1,9) = 3.86, p = 0.08$), but no significant interaction between time and experimental condition ($F(5,45) = 1.35, ns$). These data are shown in Figure 11.

Discussion

The analyses suggest that a subtle belief endowment may have real consequences. Initial wagers, which were probed for before the start of the races as is done in real-world betting markets, show significant differences between groups. As expected, participants who received tips showed an upward bias in initial wagers in favour of the tipped horse. More interestingly, participants who received an instruction to “like” a horse to win also showed an upward bias of a similar magnitude, despite the lack of apparent value in the instruction. This finding endured despite feedback to the true worthless nature of the endowment and persisted in Race 2. Experimentally-induced prior beliefs with no information value had an enduring impact on subsequent beliefs despite the presentation of new valuable data, and this behaviour looked no different from experimentally-induced prior beliefs with high information value.

Although such initial behaviour may be considered irrational by many theories of decision making, the groups’ behaviours while observing evidence provide an even more rigorous test of the impact of endowments. Once the race has begun, participants observe meaningful evidence about the horses’ likelihoods of winning. For those participants receiving a tip, the first wager made after the presentation of evidence may reasonably reflect both their endowed belief and the observed evidence; however, for those participants instructed to “like” a horse, normative theories of decision making find that to continue to consider such a prior belief in a subsequent wagering decision is irrational. Empirically, an empty “like” endowment is being utilised as if it had value even when integrated with observed evidence. The source of a prior belief—an expert, a previous experience, or perhaps even an advertisement—may not be considered in the updating process. Such a result may support coherence-based updating whereby the individual seeks to maintain a coherent set of beliefs despite any contrary evidence (Harman, 1986; Simon & Holyoak, 2002).

Wagers made during the race indicate that these biases may diminish with the accumulation of evidence as if the initial beliefs continued to be evaluated. All participant groups in both races updated beliefs based on the incoming evidence in a similar way, showing no significant differences over time. The three groups' wagers converge. Although the content and value of the belief endowments varied, the evidence suggests that the participants did not interpret evidence differently based on the endowed beliefs. The tie-break data in Race 1, however, provide a straightforward test of whether prior beliefs may still have a residual value: when both horses are tied, only participants who like the horse still show a significant bias compared to controls observing the same evidence. It may be that, while a belief based on an explicit reason may be dismissed, a "like" feeling is not as easily shaken.

A separate but still relevant issue to this discussion is introduced by the numeracy factor. The numeracy score's significant effect on predicting wagers suggests that either the less numerate use wagering to express preferences rather than maximise reward or that they are more susceptible to manipulations of beliefs.

Prior beliefs are hypothesised to have significant influence on our learning and decision making but precisely how they influence our decisions is poorly understood. Many rational accounts of learning, such as Bayesian updating, assume that decision makers evaluate their priors and weight them against the evidence in a deliberate manner. However, underlying this supposition about updating is an additional assumption about how prior beliefs are evaluated. This experiment shows that priors may not always be assessed; the weight assigned to a prior in learning may be determined by a factor such as attention rather than information value. If individuals do eventually overcome an initial judgment bias, the improvement may be narrow and unrelated to general learning or expertise.

Study 7

Playing the Hand You've Been Dealt

The game of poker is one of incomplete information and dynamic uncertainty. In addition to the randomisation of the cards as seen in previous games discussed in this thesis, there are the added elements of uncertainty from the incomplete information of opponents' cards and the unknown and changing strategic play. Indeed, it is one of only a few types of gambles that involve skill⁷. Scientists of artificial intelligence have studied the game for optimal strategies but have yet to devise a machine that can play as well as a human even in simplified versions of poker (Baker & Cowling, 2007; L. Barone & While, 1999; Billings et al., 2003; Schlicht et al., 2010).

A decision maker playing poker studies his opponents' background and behaviours for cues to decipher true abilities and likely strategies but this process, as with racetrack betting, involves filtering through large volumes of data of different sources and diagnosticities. In a previous study on decision and inference in poker, Lopes (1976) shows that the likelihood that a hand will win and the amount wagered on that bet is a multiplicative function of the subjective likelihoods of beating each individual opponent's hand. However, her experimental design removed the sequential nature of evidence presentation and play and isolated the cards from any conceptualisations about the players beyond simple labels like "conservative". It is still unknown how gamblers integrate their own prior beliefs with evidence in wagering over time.

Biased evaluations of the diagnosticity of evidence

As individuals take in new evidence over time, whether through passive observation or active search, they engage in belief updating. A gambler's beliefs about the game, including his own likelihood of winning, is constantly revised as he observes his opponents receiving cards, making bets, or even simply looking around the table at opponents' faces (Schlicht et al., 2010). However, these signals are noisy and challenging to interpret given the prevalence of bluffing and strategic play.

⁷ Whether Texas Hold'em poker is a skilled game is currently under legal debate. Analysis of 103 million hands played on the online poker site PokerStars found that 75.7% of hands ended before showdown. Of those hands that did go to showdown, 87.3% of hands that won were inferior to opponents'. These data suggest the use of skill in playing poker (Hope & McCulloch, 2009).

The ambiguous evidence gleaned from observing play is highly vulnerable to influence from prior beliefs. Hallmarks of the confirmation bias may be found as early in hypothesis testing as information search (Klayman & Ha, 1989; Wason, 1960). As a consequence, evidence that lowers the likelihood of the focal hypothesis being true or even disconfirms it may not be considered (Klayman & Ha, 1987). Evidence for alternative hypotheses may be equally or more compelling but again may not be adequately considered (Mehle, Gettys, Manning, Baca, & Fisher, 1981; Tversky & D. J. Koehler, 1994). Particularly in social scenarios, the motivation to maintain beliefs may serve to discount or misinterpret ambiguous evidence (Gilovich, 1983; Higgins, Rholes, & C. R. Jones, 1977). Consider the experiment by Darley and Gross (1983): participants observed the same video of a child taking an academic test and were asked to judge the child's academic abilities. One group of participants received no information before the viewing, another group was told the child had a high socioeconomic background, and a third group was told the child had a low socioeconomic background. While the control group assessed the child as performing at approximately grade-level, the other groups gave strikingly different responses. Those who were told the child had a high socioeconomic background judged the child to have performed well and those who were told the child had a low socioeconomic background judged the child to have performed poorly. Moreover, both groups used the same evidence to support their judgments. Prior beliefs and stereotypes may influence the processing of ambiguous evidence when assessing a person's abilities.

Despite the plethora of evidence illustrating inaccurate information search strategies and conclusions as consequences of confirmation tendencies as summarised here and in Chapter 3, the bias itself may not always be misguided. When relevant hypotheses are used to guide information search, the decision making process can be greatly accelerated and improved as the appropriate sample space is identified and explored more quickly and thoroughly. With the aid of confirmation tendencies, decision makers may efficiently converge on a likely, well-corroborated hypothesis and then try to falsify it (Mynatt, Doherty, & Tweney, 1978). In studies of poker, stereotypes of common strategies have been used effectively to improve the learning time of poker playing bots, a finding that is likely to also be descriptive of human decision making in poker (Layton, Vamplew, & Turville, 2008).

Order effects in the presentation of evidence

The likelihood of obtaining a likely, well-corroborated hypothesis may actually be an artefact of superficial constraints such as the order in which evidence is presented. Due to the intrinsically sequential nature of information processing, evidence presented early in the sequence may be used when processing evidence presented late in the sequence but evidence presented late in the sequence may dominate judgments that are made soon after the sequence (Anderson, 1981; Nisbett & Ross, 1980). In a comprehensive review of order effects in belief updating, Hogarth and Einhorn (1992) discuss order-effect phenomena in the context of their belief-adjustment model. It was found that the specific order-effect experienced in different scenarios is dependent on the type and timing of response elicited and whether information is processed online or globally.

Particularly relevant to this thesis, there is some indication that beliefs under uncertainty are affected by the primacy effect, whereby the decision making agent initially develops a belief that is then adjusted conditional on subsequent evidence. For example, Peterson and DuCharme (1967) asked participants to sample chips in a sequence and estimate the likelihood the chips sampled came from one of two urns with different colour distributions. The sequence was manipulated in the experiment such that the first one-third of trials favoured one urn, the second-third of trials the other, and the final trials were neutral. The researchers found that participants tended to favour whichever urn was indicated by the first sampling of observations and this belief was not discounted by subsequent evidence.

Of note, however, is the contrast between the model-based account of decision making described above and a rational normative account. In the task used in this study, participants are asked to complete a judgment task that requires online updating for wagering as well as global updating for learning from cues about latent factors that conflict with their prior beliefs. A model-based account of learning would predict that the order of presentation may affect the direction of attention and the generation of alternative hypotheses. An agent faced first with evidence that disconfirms his expectations may quickly turn his focus to generating alternative hypotheses while an agent faced first with evidence that supports his expectations may block attention to peripheral learning cues. The different presentation orders may result in different levels of learning from the cues in the environment. In contrast, it is predicted that the

Bayesian agent's learning and decision making would be unaffected by manipulations of order of evidence because of his ability to retrospectively assess data globally and update a distribution of priors.

A third theory makes still different predictions: recency effects, whereby the learner evaluates only evidence presented recently and therefore only indirectly integrates his initial beliefs in decisions, also predicts sensitivity to order of presentation. However, the pattern predicted for this method of learning is distinct from a model-based account because the sensitivities to incoming evidence will be in the opposite direction: while the model-based account predicts that those participants will favour initial evidence, the recency account predicts that those participants will favour later evidence.

Overview of experiment

This study manipulates the expected diagnosticity of cues in the environment and the presentation order of evidence in a game of poker. Participants watched eight hands of poker played between two computer players and were asked to learn which of two players was better, using information in the environment and from players' actions, while also wagering on the outcomes of hands. The design required the diagnostic validity of poker player characteristics to be manipulated for each individual participant on the basis of their subjective prior beliefs reported at the start of the task; the cues that participants initially believed were important were designed to be nonsignificant, while the cues that participants initially believed were nonsignificant were in reality predictive. This diagnostic validity manipulation was expected to influence how participants wagered. Additionally, the presentation order of evidence was hypothesised to affect learning of the true diagnosticity values of the cues.

Method

Design and materials

This experiment tests two hypotheses about learning and decision making over time. It was predicted that learning, as assessed by changes in cue ratings, would vary based on the order of evidence presented such that the adjustments to initial ratings by participants who saw evidence disconfirming their incorrect initial beliefs earliest would be larger compared to those who saw the same evidence later. It was also predicted that the differences in presentation order would result in different patterns of wagering, whereby

participants who first received evidence disconfirming their incorrect initial beliefs would more quickly converge to optimal wagers in favour of the truly better player than participants who received the same evidence later.

Designing the diagnosticity of cues based on prior beliefs—By collecting and using subjective prior beliefs to design each individual’s stimuli, all participants experienced a similar task regardless of their knowledge about and experience with poker. To elicit subjective prior beliefs about poker players, ratings of the importance of different cues were collected.

Profile items included a range of characteristics (e.g., number of years experience playing poker, number of languages spoken, favourite number). At the start of the task and again at the end of the task, participants were asked to rate the nine characteristics on how likely the better player of the two being observed in the task would be to have that characteristic. Ratings questions probed for magnitude of importance and the direction of the relationship; for example, participants were instructed to move the response bar to the right if the better player was likely to have the characteristic and further to the right if the player was more likely to have that characteristic. A response in the middle of the scale indicated the participant thought the characteristic was irrelevant to poker player quality. When rating items at the start of the task, response bars began in a default position in the middle but when re-rating items at the end of the task, response bars were shown in the positions left by the participant in the earlier response stage.

The logic used to create player profiles is illustrated in Table 2. The profiles of the poker players were limited to five items, or cues. Of the five characteristics displayed, two were items that had been ranked as highly predictive by the participant (ranked second and third out of nine in absolute magnitude of importance) and designed to match the participant’s expectations of a high quality player. For example, if a participant indicated that playing professionally was highly likely to be associated with the better player, one poker player would be explicitly labelled as “professional” in his profile while the opponent labelled as “amateur”. A third displayed characteristic had been ranked of medium importance by the participant (ranked fifth out of nine) and was designed to favour the inferior player; the “professional” player looked relatively worse on this item. The fourth and fifth characteristics displayed were the fourth ranked item (designed such

that the two players were equally matched) and the eighth ranked item (designed in favour of the “professional” player). Ultimately, one poker player excelled on dimensions that the participant indicated as being highly predictive of a good poker player while the other poker player looked relatively poor and only scored highly on one dimension, which was of medium importance.

However, actual performance during the eight hands played in the task clearly favoured the player that the participant expected to be worse. The “better” player made severe mistakes representative of a poor player. Therefore, only one cue, which had been rated of medium importance (Cue D), effectively predicted overall player quality.

Table 2. Cues were displayed for the poker players’ profiles based on each participant’s initial ratings of prior beliefs. Player profiles were created on an individual basis to match each participant’s subjective expectations of high and low quality players, thus ensuring that each participant experienced a similar task. A + indicates that the player is superior on that dimension, a – indicates inferiority, and an = indicates the players are equally matched.

<i>Initial Item Rank</i>	<i>Profile Items</i>	<i>“Better” Player Profile</i>	<i>“Worse” Player Profile</i>
1 st			
2 nd	Cue A	+	-
3 rd	Cue B	+	-
4 th	Cue C	=	=
5 th	Cue D	-	+
6 th			
7 th			
8 th	Cue E	+	-
9 th			

Manipulating presentation order of evidence—The stimuli presented were designed to either confirm or disconfirm the participant’s initial belief about which player was better. Evidence that strongly disconfirms the participant’s initial beliefs shows one player, the player that the participant initially favoured, make severe mistakes. Four hands showed consistent disconfirming evidence. For example, the player folds despite having a strong hand. The other four hands provide evidence that weakly supports the participant’s initial beliefs: the initially favoured player performs at an adequate level, providing no strong positive or negative evidence. During all eight hands, the other player, expected to be inferior, plays adequately and on balance performs at a higher level. For example, one player bets after cards that are favourable for him come out on the flop and the other player folds on what seems like a weaker hand; both players perform adequately in a

straightforward hand that does not provide strong positive or negative evidence about the quality of either player. Participants in one experimental group observe first the block of evidence that disconfirms their initial beliefs and then second the block of evidence that supports their initial beliefs, while the second experimental group observed the two blocks of four hands in the reverse order. It was emphasised in the task instructions that the hands were not shown in the order that they had occurred so that participants would not infer causality or expect carryover effects between hands. An ideal agent in this paradigm would evaluate each hand independently.

Before each hand, participants were asked to make wagers on which of the two players would win the next hand. Participants allocated £10 virtual money to the two players; participants were instructed to place £5 on each player if they thought each was as likely to win as the other or more or all of the £10 on the player they thought was more likely to win. After wagering, each hand was played out.

Participants were randomly assigned to an experimental condition and remained unaware of any alternative task specifications. In each condition, participants were informed that their compensation would depend on the outcomes of the decisions and choices they made in the task. All participants read the same instructions regardless of experimental condition.

Participants

Thirty-nine subjects were recruited to participate in a paid study on gambling from the University College London Psychology Department Subject Pool, Gumtree (a popular online UK notice board), and local newspapers. Participants were randomly assigned to conditions. Several participants (15% of the sample) were removed because their response pattern suggested a misunderstanding of the response scale, resulting in a final sample of 19 participants in the group observing disconfirming evidence first and 14 participants in the group observing the evidence in the reverse order. In self-reported ratings of poker knowledge and experience, the sample responded with a mean 2.31 (SE = 0.53) out of 10 rating for poker knowledge and mean 2.75 out of 10 (SE = 0.63) for poker experience.

Procedure

This experiment was run in conjunction with three unrelated experimental tasks on gambling. The tasks were administered in the same order for all participants; this experiment was run fourth in the series. It is assumed there are no effects of the other experimental tasks.

At the start of the task, participants were seated at a computer with instructions for the task. Participants were instructed they would be viewing, but not playing, a selection of eight hands played by two players in a no-limit Texas Hold'em online tournament that had occurred in the past. Participants would have complete access to the players' cards and bets but not the players' strategies, intentions, or quality information except for a brief profile.

The instructions also included a brief tutorial on the basic rules and strategies of no-limit Texas Hold'em. Participants were advised that both the dealt cards and also how the poker players bet against each other were important in winning a hand; taking advantage of opportunities to bluff (bet a large amount on a weaker hand to encourage the opponent to fold) and value-bet (bet a small amount on a stronger hand to encourage the opponent to call) may be indicators of the quality of a player and the likelihood that he will win a hand. Due to the brevity of the tutorial and the anticipated variance in prior poker knowledge, summaries were shown after each hand that concisely described how each poker player had played in that hand.

After reading the instructions, participants first rated profile items to report their prior beliefs about what characteristics they associated with good poker players. Unbeknownst to the participants, this rating information was then used to design the remainder of the task. Immediately after rating profile information, participants were shown the profiles of the two fictitious players and prompted to wager on which player would win the next hand, and then the hand was played out. This wagering procedure was repeated for eight hands, with the profiles displayed at each wagering prompt. When the eight hands were finished, participants were prompted to re-rate the profile information.

Results

In this experiment, a between-groups design tested the effect of order of presentation of evidence on how participants learned about the diagnosticity of cues and made decisions in a dynamic environment of uncertainty. Participants made two types of judgments: profile item ratings and wagers. It was predicted that participants who first observed evidence that disconfirmed initial beliefs would attend to peripherally diagnostic cues earlier, resulting in superior learning and optimal wagering compared to participants who observed evidence in the reverse order.

Pre-task ratings of cue importance

Ratings elicited at the start of the task represent participants' prior beliefs about poker players. Participants were asked to rate nine items of information on how likely the better player of the two being observed would be to have or represent the characteristic described. Mean ratings, shown in the second column of Table 3, suggest that the participants had a basic understanding of characteristics associated with poker player quality. As expected, ratings between experimental groups were not significantly different (all p values > 0.06 , except the ninth cue, $p = 0.005$, though this is still greater than the multiple-tests adjusted threshold). Group differences between those with some degree of poker knowledge (self-rated score greater than or equal to 2; $n = 14$) and those with little to no knowledge ($n = 19$) were also non-significant (all p values > 0.22).

Post-task ratings of cue importance

After the task of viewing poker hands was completed, participants were prompted with the opportunity to adjust their original pre-task ratings. Mean post-task ratings are shown in the third column of Table 3. As expected, there were no significant differences between groups on these raw ratings (all p values > 0.16) because participants within groups were shown different cues conditional on their prior beliefs. Group differences between those with some degree of poker knowledge and those with little to no knowledge were again non-significant (all p values > 0.16).

Changes in ratings

To evaluate how participants learned during the task, the difference in ratings collected before and after was measured. The simple difference between the two ratings including their original valence was used so as to measure increase in likelihood that the

characteristic is associated with the better player; the participants' rating scale ranged from "very unlikely" to "very likely". Although this calculation neglects certain aspects of information gain, it was believed to be the most relevant interpretation for the present hypotheses. Because the cues seen by each participant varied, the data are analysed and shown as cues rather than specific profile items.

Table 3. Participants rated nine items of information on how likely the better player of the two would be to have each characteristic using a sliding bar. The response scale ranged from -50 to +50 (no labels appeared on the screen), where a response of +50 suggests the participant believed the item was very likely to be associated with the better player and -50 suggests the participant believed the item was very likely to be associated with the worse player. Initial ratings were collected before the start of the task and final ratings were collected after viewing all hands played.

<i>Profile Item</i>	<i>Mean Initial Rating (SE)</i>	<i>Mean Final Rating (SE)</i>
He has more years of experience playing poker than his opponent	32.97 (2.31)	18.76 (2.84)
He plays poker more professionally (rather than as an amateur) than his opponent	32.15 (2.69)	24.03 (3.01)
He won more money in this same tournament last year than his opponent	14.33 (2.91)	13.88 (2.96)
He eats more before tournaments than his opponent	-8.45 (2.80)	-7.03 (3.17)
He participates in more sports than his opponent	4.24 (2.59)	2.73 (3.19)
He speaks more languages than his opponent	-3.42 (3.12)	-4.52 (3.39)
He has a higher favourite number than his opponent	-3.03 (2.66)	-0.33 (3.28)
The player gambles on more non-poker games than his opponent	-1.94 (2.65)	-1.72 (3.05)
He has more siblings than his opponent	1.61 (2.82)	-0.48 (2.78)

Of particular interest to the present hypotheses, are changes in ratings for Cue A, a profile item that the participant had initially rated as highly predictive of poker player quality and that did not match the subsequent evidence, and Cue D, a profile item that the participant had initially rated as a poor predictor of poker player quality and that did not match the subsequent evidence.

Updating of a focal cue—In post-task ratings of Cue A, participants adjusted their ratings of importance downward in agreement with the overall sum of evidence, as confirmed by a t-test against 0.00 ($t(32) = -5.69, p < 0.001$), shown in Table 4. Analysis shows that the

order in which evidence was presented did affect the participants' judgments (Confirm-Disconfirm: $M = -17.97$, $SE = 3.41$; Disconfirm-Confirm: $M = -9.95$, $SE = 2.81$; $F(1,29) = 3.30$, $p = 0.08$, $\eta^2_p = 0.14$), despite the power to detect this difference being low (power = 0.42). This analysis also showed that there was a significant interaction with level of reported poker knowledge ($F(1,29) = 4.72$, $p = 0.04$, $\eta^2_p = 0.14$, power = 0.56), which is illustrated in Figure 12. In other words, there were no order effects for participants with no or little prior knowledge of poker but participants already familiar with poker did report a changed rating. The data suggest that the more knowledgeable participants' beliefs based on focal cues were highly sensitive only if their strong prior beliefs were *recently* disconfirmed. The main effect of poker knowledge was not significant ($F(1,29) = 0.15$, ns, power = 0.07).

Table 4. Mean changes in cue ratings after viewing profiles and all hands played.

<i>Profile Item</i>	<i>Initial Item Rank</i>	<i>Mean Initial Rating (SE)</i>	<i>Mean Final Rating (SE)</i>	<i>Mean Difference (SE)</i>
Cue A	2 nd	21.58 (4.95)	9.15 (3.91)	-12.79 (2.25)
Cue B	3 rd	9.48 (4.37)	11.12 (4.12)	-1.88 (1.59)
Cue C	4 th	4.06 (3.75)	1.21 (3.80)	-1.39 (1.71)
Cue D	5 th	3.55 (2.65)	4.58 (3.32)	2.30 (1.63)
Cue E	8 th	0.36 (1.44)	-0.91 (2.81)	3.88 (1.75)

Updating of a peripheral cue—In post-task ratings of Cue D, participants did not adjust their ratings of importance upward in agreement with the overall sum of evidence, as confirmed by a t-test against 0.00 ($t(32) = 1.42$, ns), as shown in Table 4. However, analysis shows that the order in which evidence was presented did affect the participants' judgments (Confirm-Disconfirm: $M = 8.22$, $SE = 2.19$; Disconfirm-Confirm: $M = -0.57$, $SE = 1.80$; $F(1,29) = 9.60$, $p < 0.01$, $\eta^2_p = 0.25$); those who viewed evidence disconfirming their initial beliefs last reflected the evidence while those who viewed disconfirming evidence first did not, contrary to predictions. Poker knowledge also had a main effect on judgments, whereby those with some or great prior knowledge adjusted their ratings upward significantly more than those with no or little prior knowledge (Unknowledgeable: $M = -0.18$, $SE = 1.80$; Knowledgeable: $M = 7.83$, $SE = 2.19$; $F(1,29) = 7.97$, $p = 0.01$, $\eta^2_p = 0.22$). Again, the interaction of these two factors was also significant ($F(1,29) = 3.82$, $p = 0.06$, $\eta^2_p = 0.12$, power = 0.47), as shown in Figure 12. These data suggest that the more knowledgeable participants who viewed disconfirming

evidence early quickly discounted it in favour of the confirming evidence that came after, while knowledgeable participants who viewed disconfirming evidence late easily discarded their initial expectations.

Knowledgeable participants who observed first confirming evidence and later disconfirming evidence demonstrated superior learning from both focal and peripheral cues. Unknowledgeable participants appear to have learned only about focal cues. A mixed ANOVA with factors of condition, poker knowledge, and cue type finds that the three-way interaction is marginally significant ($F(1,35) = 3.77, p = 0.06, \eta_p^2 = 0.10, \text{power} = 0.47$).

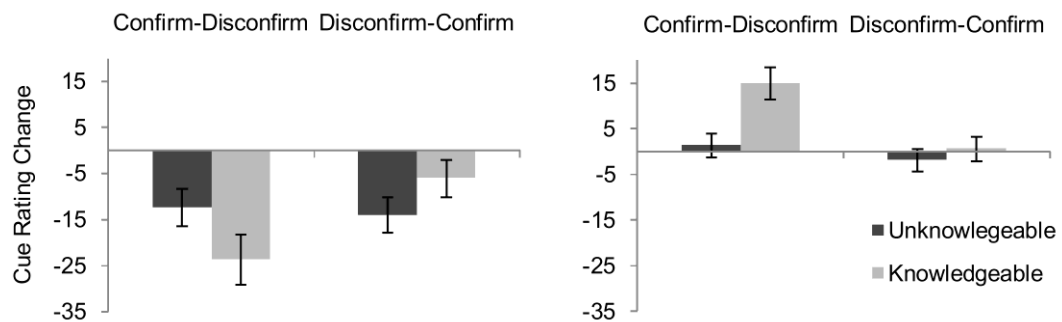


Figure 12. Change in participant ratings of cue predictive strength. The panel on the left shows data for Cue A, an item initially ranked as having high importance. The panel on the right shows data for Cue D, an item initially ranked as having medium importance. Change was calculated by taking the difference of the absolute means of each group. Bars show standard errors of the means.

Wagers

Participants were prompted before the start of each hand to wager on which player they believed would win the next hand. As shown in Figure 13, all participants initially favour the “better” player that matched their expectations for a high quality poker player to a similar degree (Confirm-Disconfirm: $M = 6.86, SE = 0.51$; Disconfirm-Confirm: $M = 6.00, SE = 0.45$; $t(31) = 1.25, ns$). And, participants’ wagers on their respective eighth and final hands are also not significantly different (Confirm-Disconfirm: $M = 4.86, SE = 0.61$; Disconfirm-Confirm: $M = 4.79, SE = 0.50$; $t(31) = 0.09, ns$). As participants initially process only one type of evidence (either confirming or disconfirming), there are significant differences in wagers (time periods one through four), but as the second type of evidence is integrated with prior beliefs (time periods five through eight, including the evidence observed up to this point) the experimental groups’ wagers converge.

To understand the effects of the different types of evidence on wagering, we compare the two experimental groups' wagers from different chronological time periods when the groups viewed the same evidence. Because of the predictive nature of the wagers, only three wagers out of four from each evidence block are comparable across experimental groups (i.e., one group's first wager for the block of confirming evidence is made based on prior beliefs only while the other experimental group's fifth wager for the block is based on the previous block of four hands of disconfirming evidence shown as well as their prior beliefs). This is shown in Figure 14.

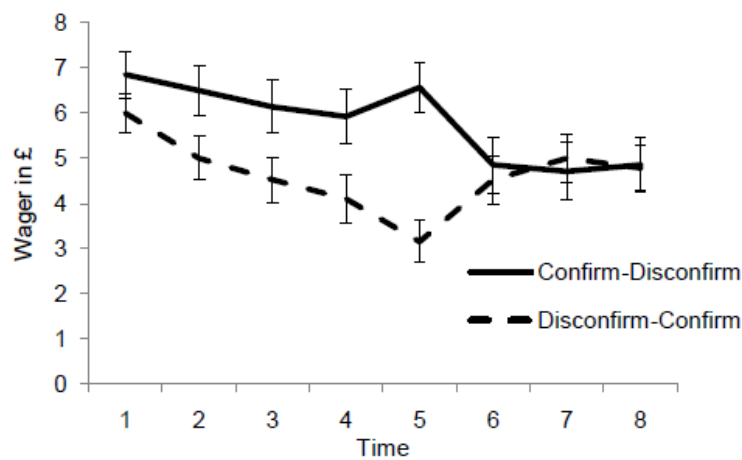


Figure 13. Mean online wagers for the upcoming hand in favour of the “better” player, with chronological time on the horizontal axis, are shown here. Bars show standard errors of the means.

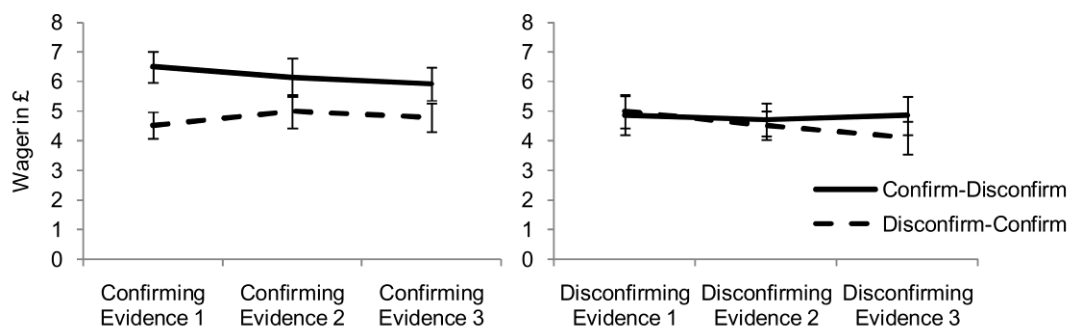


Figure 14. Mean online wagers for the upcoming hand in favour of the “better” player, with type of evidence preceding the prediction on the horizontal axis, are shown here. In the left panel, data represent the wagers made after viewing evidence that confirms participants' initial beliefs; in the right panel, wagers made after viewing evidence that disconfirms participants' beliefs. Bars show standard errors of the means.

When processing confirming evidence, there was a significant difference between experimental groups: after observing evidence that disconfirmed their beliefs about the “better” player, participants wagered significantly less in favour of the “better” player (mixed ANOVA $F(1,31) = 4.81, p = 0.04, \eta_p^2 = 0.13$). This difference was not significant when processing disconfirming evidence ($F(1,31) = 0.15, ns$); first observing disconfirming evidence did not lower participants’ wagers when observing evidence aligned with expectations. Adding poker knowledge as a covariate did not improve the model.

Discussion

It was hypothesised that presentation order of evidence that confirmed and disconfirmed participants’ prior beliefs about poker player quality would have a significant effect on wagering and learning about cue diagnosticity in the experimental task. Pre- and post-task ratings of cue importance were compared to assess learning as well as wagers collected online during the task. Learning was found to vary on two factors: presentation order and self-rated knowledge of poker. Wagers reflected a decreasing confidence in initial expectations with final wagers approximating indifference between the two players regardless of experimental condition.

Learning from cues

For both subjectively focal and peripheral cues, knowledgeable participants exhibited recency effects in learning and wagering. This pattern of results does not support a Bayesian learning prediction (no presentation order effects) or a model-based learning prediction (primacy effects). The data may indicate that participants with expertise completed the task differently. These participants may have been more sensitive to the incoming evidence than to the profile information given (several items of which were nondiagnostic, by design) and ultimately judged players by their observable decisions. Also, these participants may have treated the experimental task as a typical poker game, and made inferences about poker player quality from the change in play quality over time.

The data suggest that unknowledgeable participants also show recency effects but that they are less capable of learning from the task; unknowledgeable participants learn from salient focal cues only and block all learning from peripheral cues in the environment.

This finding suggests that lay participants may weigh their prior beliefs heavily compared to the evidence observed. Despite being unknowledgeable about the game, these participants may believe their prior beliefs to be more reliable evidence than their naive interpretations of the incoming evidence. The pattern of results would be explained by a relatively high confidence (weight) in the strong prior beliefs (Cue A) but no confidence in either the weaker prior beliefs (Cue D) or the processing of incoming evidence.

Overall, the relatively strong response by all participants to the focal cue may also be a result of the relatively strong evidence used to disconfirm it in the experimental design: the poker player made severe mistakes. In contrast, the response to the peripheral cue was based on relatively weak evidence used to confirm it in the experimental design: the players' performance was not equivalently positive. Direct comparisons between ratings of the two cues should be made with careful consideration.

Wagering

The wagering data show that participants are highly sensitive to disconfirming evidence. When observing evidence that disconfirmed prior beliefs, participants reduced wagers in favour of the player regardless of previous evidence observed. However, when observing confirming evidence, the disconfirming evidence still weighed heavily into wagering decisions. However, this result may also be a product of the above mentioned disparity in experimental design of evidence strength. Regardless of this limitation, the convergence of participants' wagers across experimental groups indicates that presentation order does not significantly affect decision making. Similarities may be drawn to jury decision making, where jurors must process evidence presented in non-chronological order from both the prosecution and the defence. The findings in this poker study suggest that jurors' decisions may also not be a function of presentation order of evidence. Currently, there is mixed evidence on this issue (Kerstholt & Jackson, 1998; Lagnado & Harvey, 2008; Pennington & Hastie, 1992).

Expertise

The analyses of the effect of expertise on learning support the literature on expert confirmation bias in games. In chess games, experts have been found to exhibit the confirmation bias (Bilalic, McLeod, & Gobet, 2008; Cowley & Byrne, 2004). Of particular relevance to the present findings, Bilalic, et al. (2008) concluded that this

occurs by influencing mechanisms that determine what information is attended to. Eye tracking confirmed that experts persisted in focusing on the first hypothesis while ostensibly searching for alternatives. The present data show that not only do agents attend to those cues during information search and decision making, but they also store the information in this way, in some instances neglecting peripheral information.

The full process of belief updating includes first attending to evidence, then evaluating it, and finally combining it with prior beliefs. The data presented here illustrate how decision makers including experts may deviate from optimal standards. Particularly in environments that conflict with prior expectations, the updating process determines how quickly an agent converges on a more appropriate model of the environment. In this experiment, it was shown that the coherence bias exhibited in Study 6 of this chapter may be due to biases in attention whereby peripheral cues are not attended to. Although the evidence does not support the model-based account's strong predictions, the significant interactions between knowledge and presentation order do suggest a variation of a model-based account while at the very least casting doubt on the usefulness of the normative and heuristic approaches. Whereas rational theories of decision making assume that the process is isolated from the limitations and disruptions that are often found elsewhere in cognition, such as attention blind spots, confirmation biases, and working memory limitations, the present study demonstrates the importance of understanding prior beliefs and the biased nature of belief updating under dynamic uncertainty in gambling.

Chapter 5

General Discussion

My goal for this thesis was to present a strong case to researchers to study gambling from a model-based perspective. To support my case, I drew upon the established findings of several lines of research including real-world and laboratory-based gambling studies, inductive inference, heuristics and biases, reinforcement learning, hypothesis evaluation, and mental models. Despite the dominance of other perspectives in contemporary gambling research, I argued that the model-based approach made a superior fit with gambling behaviour because it simultaneously captures the realistic strengths and weaknesses of cognition found in empirical research while also making sound theoretical sense. I continued by presenting seven studies, which ranged from simple risk to dynamic uncertainty, laboratory to real-world contexts, and computational modelling to abstract reasoning. By covering such a broad range, I intended to show the breadth and depth of the model-based approach.

Summary

In Chapter 2, we studied simple games of risk. By dismissing the traditional assumption that punters relied solely on the incoming evidence, it became evident that a significant portion of punters do just the opposite. Similarly, by altering the framing of a dice game, it was found that beliefs are sensitive to perceived (and not formally relevant) representations of how the game works. The studies illustrated that individuals use internal representations of games, which include beliefs about the underlying outcome-generating process.

In Chapter 3, to examine the cognitive processes underlying these behaviours, we studied slot machines, a game of fixed but unknown uncertainty. The first two studies demonstrated that people consider prior knowledge when updating beliefs, resulting in hypothesis testing biases in inference that persist despite the absence of diagnostic confirming evidence. And crucially, we also saw that beliefs about outcome space affected beliefs about outcome-generating processes. To complement these findings on a computational level, a modelling exercise simulated individual decision making and found that an assumption of a similarity mechanism of judgment is insufficient for producing

the empirical data presented. Combined with the findings of Chapter 2, these results make evident the highly structured nature of people's internal representations in gambling.

In Chapter 4, we studied the process of belief revision in the dynamically uncertain paradigms of horse race and poker betting. It was found that participants seemed to prefer coherence and consistency in beliefs over time rather than maximum reward. A second experiment showed that this phenomenon was due to attentional learning biases whereby focal cues were learnt at the expense of peripheral cues. However, this result interacted with expertise such that knowledgeable participants were able to attend to all cues.

When the facts changed, what did participants do? They changed the facts. In the studies presented here, it was shown that people develop and maintain internal representations of problems that may be separate from reality, even at the expense of profits. When tasked specifically with learning about their environment, prior beliefs and expectations drove learning, regardless of what the data suggested. These studies have shown that people weigh their prior beliefs more heavily than the data, regardless of their priors' source or quality. They weigh less heavily their evaluations of incoming evidence, and further these evaluations are subject to assessments of quality or confidence unlike prior beliefs. It seems that people were less inclined to change their minds in the face of disconfirming evidence because they did not see that it was disconfirming.

Consider again the two questions I put forward at the start: why do people continue to gamble despite losses? and why do only some people have problems with only some forms of gambling? These are hard questions that will take a multidisciplinary effort to resolve. But here I have shown that part of the answer lies in misunderstanding the fundamental type of uncertainty underlying the problem and failing to see and use the true value of evidence. A framework built around the individual's prior beliefs, perception of the outcome-generating process, and updating of his beliefs is a productive way to pursue answers to these questions. Within this framework, other disciplines can take on parts of the puzzle, from anthropologists and sociologists contributing to understanding how prior beliefs are developed to computational scientists contributing to understanding how beliefs are updated. Several disciplines are already on board,

including artificial intelligence researchers (Sutton, 1991), neuroscientists (Dayan & Niv, 2008), psychologists (T. L. Griffiths & Tenenbaum, 2005) and economists (Feltovich, 2000).

Broader implications for cognitive psychology

In each study presented here, it was found that structured representations of the games and understanding of the underlying outcome-generating processes were critical for describing individual behaviour. This thesis supports research in the greater field of cognitive psychology on the role of causal models. For example, the findings here echo the arguments in Krynski and Tenenbaum (2007); in this probabilistic reasoning framework, deviations from normative models are explained by Bayesian inferences over causal models. The authors show that people's judgments and reasoning are related to the structure of their mental representations. And these results also support recent findings on the importance of probabilistic causal models in evidential reasoning (Lagnado, 2010; Sloman, 2005). The evidence presented in this thesis and in an emerging collection of research indicates that descriptive accounts of decision making and reasoning must make room for structure and causal models.

Future directions

The results presented here indicate that there are many applications and extensions of the model-based approach to studying gambling. As with any study of gambling, further replications of these results in real-world contexts with significant monetary gains and losses and, of course, the option to not play at all, would add a solid foundation of support for the validity of the data.

The development of methods for eliciting prior beliefs from individuals would be an interesting and practically useful next step for this research. As we have seen, individuals maintain rich representations of problems that affect how subsequent evidence is interpreted and judgments and decisions are made. By emphasising the importance of structured representations, researchers can understand how representations of events and abstract concepts and categories are structured and related to each other. The graphical methods [hand-drawn pie charts] used in Chapter 3 Study 4 to elicit subjective probabilities about potential outcomes is one step in this direction. Indeed, conducting this work in the gambling domain should lead to interesting developments due to the

nature of randomness in games and hidden outcome-generating processes. Refining this method may lead to multi-disciplinary use and accurate and standardised collection of subjective probabilities for use in understanding how subjective probability judgments relate to actual probabilities in gambling and in other areas of research.

To advance the field on a different level, process-level models may be developed based on the initial work presented here. There are already researchers developing models for learning of causal schemas in everyday reasoning using hierarchical Bayesian frameworks (Kemp, Goodman, & Tenenbaum, 2007). Future work in developing a computational account of gambling may use these as foundations for a model-based account.

Impact of the results for users of gambling research

The approach advocated in this thesis creates a space for policymakers and clinicians to take on gambling research anew. Although the studies presented here primarily used non-gamblers, where comparisons were made to regular gamblers there was no evidence of significant differences. This notion is supported by previous literature (Corney & Cummings, 1985; Delfabbro, 2004; Wagenaar, 1988). It may be helpful for the field of gambling research to recognise that “gamblers” are not so different from others, and gambling is not so different from other tasks in everyday life.

Policymakers might take away that the greatest impact that policy can have on gambling problems is the minimisation of the prevalence of chance elements in games. Even with games as simple as roulette, where the outcomes and their probabilities are transparently available and the outcome-generating process is known, people find ways to impose subjective control over uncertainty. Increasing transparency, improving signage, and reducing marketing efforts are all small steps toward reducing the outward appearance of gambling harm but may have no significant effect on the prevalence and development of gambling problems. Indeed, the proportion of problem gamblers in the UK has held steady for the last ten years (National Centre for Social Research, 2007).

Clinicians should see that this research reflects much of their own work in understanding gambling behaviour at the individual level but might also take away that current methods of cognitive-behavioural treatment and statistical training may be updated to include elicitation of prior beliefs before determining a course of treatment. There is no single

irrational mindset that fits all those with gambling problems. By working with the patient's internal representations and hypotheses about the way that the game works, the type of evidence that is able to disconfirm inaccurate beliefs and change minds may be found.

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Appendix 1
 Problem Gambling Severity Index (Ferris & Wynne, 2001)

Thinking about the last 12 months...

	Never	Some of the time	Most of the time	Almost always
Have you bet more than you could really afford to lose?				
Have you needed to gamble with larger amounts of money to get the same feeling of excitement?				
When you gambled, did you go back another day to try to win back the money you lost?				
Have you borrowed money or sold anything to get money to gamble?				
Have you felt that you might have a problem with gambling?				
Has gambling caused you any health problems, including stress or anxiety?				
Have people criticized your betting or told you that you had a gambling problem, regardless of whether or not you thought it was true?				
Has your gambling caused any financial problems for you or your household?				
Have you felt guilty about the way you gamble or what happens when you gamble?				

Scoring Instructions for the PGSI

The total score is calculated using the following scale:

Never = 0

Some of the time = 1

Most of the time = 2

Almost always = 3

Scores for the nine items are summed, and the results are interpreted as follows:

0 Non-problem gambling.

1 - 2 Low level of problems with few or no identified negative consequences.

3 - 7 Moderate level of problems leading to some negative consequences.

8 + Problem gambling with negative consequences and a possible loss of control.

Appendix 2

Numeracy Scale (Lipkus, Samsa, & Rimer, 2001)

Imagine that we rolled a fair, six-sided die 1,000 times. Out of 1,000 rolls, how many times do you think the die would come up even (2, 4, or 6)?

In the Big Bucks Lottery, the chances of winning a 10 pound prize is 1%. What is your best guess about how many people would win a 10 pound prize if 1,000 people each buy a single ticket to Big Bucks?

In the Acme Publishing Sweepstakes, the chance of winning a car is 1 in 1,000. What percent of tickets to Acme Publishing Sweepstakes win a car?

Which of the following numbers represents the biggest risk of getting a disease?
1 in 100 1 in 1000 1 in 10

Which of the following numbers represents the biggest risk of getting a disease?
1% 10% 5%

If Person A's risk of getting a disease is 1% in ten years, and person B's risk is double that of A's, what is B's risk?

If Person A's chance of getting a disease is 1 in 100 in ten years, and person B's risk is double that of A's, what is B's risk?

If the chance of getting a disease is 10%, how many people would be expected to get the disease out of *100*?

If the chance of getting a disease is 10%, how many people would be expected to get the disease out of *1000*?

If the chance of getting a disease is 20 out of 100, this would be the same as having a ____% chance of getting the disease.

The chance of getting a viral infection is .0005. Out of 10,000 people, about how many of them are expected to get infected?